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ABSTRACT

The development of this management system begins with an exploration of what the library is attempting to achieve. After an investigation of the hierarchy of library objectives, it is concluded that the one objective which satisfies a set of specified criteria is to maximize the expected future exposure of university community members to documents given current and anticipated future expenditures. A measure of performance utilizing the concept of item-use-days is developed to evaluate the degree of attainment of this objective. Finally, the difficulties encountered in constructing an effectiveness measure to make year-to-year or interlibrary comparisons are explored. The Planning-Programming-Budgeting System is presented as a rational framework for the management of university libraries. Library activities are classified according to four different dimensions. A typical program structure is enumerated utilizing these dimensions. Benefit-cost analysis is discussed as a means for determining an effective allocation of the library's resources. It is shown that resources are allocated and utilized in the most effective manner when the marginal benefit-to-cost ratios for all elements of all programs are equal. Guidance is also offered for the development of a statistical information system to support the decision-making and planning process. (Author/NH)

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# THE DEVELOPMENT OF A MANAGEMENT SYSTEM FOR EFFECTIVE DECISION MAKING AND PLANNING IN A UNIVERSITY LIBRARY

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December 1972

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Michael R. W. Bommer

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# ABSTRACT

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The Development of a Management System for Effective Decision Making and Planning in a University Library.
Supervisor: Morris Hamburg

This research represents an attempt to develop an improved management system to provide guidance to university library administrators for making decisions and formulating plans. This management system provides a rational framework for library administrators to effectively allocate and control scarce resources and to justify current expenditures and requests for additional funds. The system is designed to be sensitive to the differing structures and compositions peculiar to different universities so as to be applicable to any university library. Innovations in library technology should not necessitate major changes in this system. In addition, modern management techniques, in particular, analytical decision models, are readily integratable into this system.

The development of this management system begins with an exploration of what the library is attempting to achieve. After an investigation of the hierarchy of library objectives, it is concluded that the one objective which satisfies a set of specified criteria is to maximize the expected future exposures of university community members to documents given current and anticipated future expenditures. A measure of performance utilizing the concept of item-use-days is developed to evaluate the degree of attainment of this objective. Finally, the difficulties encountered in constructing an effectiveness measure to make year-to-year or interlibrary comparisons are explored.

The Planning-Programming-Budgeting System is presented as a rational framework for the management of university libraries. Library activities are classified according to four different dimensions. A typical program structure is enumerated utilizing these dimensions. Benefit-cost analysis is discussed as a means for determining an effective allocation of the library's resources. It is shown that resources are allocated and utilized in the most effective manner when the marginal benefit-to-cost ratios for all elements of all programs are equal.

Analytical models are developed utilizing techniques of Operations Research and Statistical Analysis to provide guidance to library administrators in the decision-making and planning process for selected subprograms. In the construction of these models relevant variables and factors are identified and pertinent relationships determined germane to each subprogram. The decision models are solved to yield marginal benefit-to-cost ratios which

relate to the overall measure of library performance. Decision models are constructed for determining the number of multiple copies of a title to purchase for a subject area, the number of new titles to purchase for a subject area and the number of copies of a title to place on reserve. In addition, a model is constructed to predict the demand rate for circulation use of titles as a function of the circulation use rate and measures of the teaching and research activities of the university for each subject area.

These models are then tested and verified and relevant parameters estimated as applied to a departmental library at the University of Pennsylvania. For each model an example of its use is demonstrated and a sensitivity analysis is performed.

Finally, guidance is offered for the development of a statistical information system to support the decision-making and planning process. In this regard criteria are presented which can be used to evaluate various information elements for inclusion in the statistical information system.

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# PREFACE

At present there exists a critical need for an improved management system to provide guidance to library administrators for making decisions and formulating plans. This need emerges, on one hand, from an increase in the intensity and complexity of demands placed upon the university library by a user clientele which is growing in size and intellectual sophistication. On the other hand, the number of documents published each year and the advances in library technology to provide access to the phenomenal growth of knowledge are increasing at a significant rate. At a time when demands are increasing for access to a burgeoning volume of knowledge, university libraries are expected to accommodate these trends at little increase in operating costs. Thus, the development of an effective management system to allocate and control scarce resources and to justify current expenditures and requests for additional funds is of paramount importance to the university library administration. To date little has been provided in the way of a framework for rational and systematic decision making and planning for a university library. This dissertation is addressed to this problem.

In this research the library was viewed as a system whose major responsibility is to satisfy the greatest number of demands generated by all members of the university community. This view is in contrast to the belief held by some that a library is a depository for all recorded knowledge. Certainly the preservation of certain materials, rare books for example, which might be used on a few occasions, is an important function. However, when compared with the responsibility to provide prompt access to a great volume of material for a great number of users within a budget constraint, the relative importance of this archival function appears to diminish.

The system presented in this dissertation does not negate but rather augments the research in this area to date. This system provides a framework for the employment of analytical models developed by various researchers. In addition, new analytical decision models are developed in this dissertation to provide direction to library administrators for the purchase of new titles and multiple copies of titles for circulation and reserve use.

No pretense is made that the system developed in this dissertation is either complete or readily implementable. Decision models are lacking for many facets of library operation. In addition, many details must be attended to prior to the employment of this management system. Nevertheless it is believed that the system presented here represents a basis for more effective decision making and planning.

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# CHAPTER I

# INTRODUCTION

## Need for Improved Decision Making

The university library is generally acknowledged as playing a vital role in the educational process of the university. In recent years, as the teaching and research programs of the university have become both more diverse and specialized, the importance of the library's role has increased. Most university administrators are aware that maintenance or improvement of the quality of research and instruction carried on at their universities require complementary actions in the quality of service offered by the library.

However, even to maintain the same quality of library service generally requires an increase in the level of service. At most universities the size of the university community (faculty, staff, and students) is growing, which results in increased demands for library service. The caliber of research and instruction is improving as the intellectual level of the incoming students and the level of sophistication of researchers rises, resulting in an increase in the number and complexity of demands for documents of high scholarly competence. At the same time the teaching process is evolving to encompass a broader spectrum of knowledge and to place more responsibility on students to pursue independent learning. In recent years research programs have become more considerate of the impact of new developments on all facets of humanity and the environment. These trends place a greater burden on the library for meeting these expanding needs. Finally, user expectations are increasing. Not long ago many analysts were quite happy to walk four blocks to utilize a computer with a one day turnaround. Today, a turnaround time in excess of a half-hour on a computer terminal in the next building is often considered an inconvenience. Similarly, the user of the university library expects expanded service provided in a reduced timeframe.

Not only must the library cope with an increase in the intensity and complexity of demands generated by a changing educational environment within the university, it must also cope with a proliferation of documents reflecting the exponential growth of knowledge. This proliferation is not confined to the growth of knowledge along traditional lines. New fields of knowledge are evolving and the way in which knowledge is structured within disciplines is changing. These trends dictate a continuing expansion of the library's document procurement program as well as the development of subject specialists for selecting new documents and for assisting users, if the library is to maintain its current level of quality of service.

At the same time that libraries are experiencing a growing demand for service and are being expected to provide access to an increasing volume of knowledge, educational institutions are experiencing a financial squeeze. Libraries are not exempted from this plight. The university library is expected to provide expanded services with little increase in funds. To cope with this dilemma it is imperative that university libraries allocate and utilize their scarce resources in the most effective manner to meet the needs of the university community.

## Benefits of a Management System

For years the scientific management approach has been employed successfully in production oriented organizations in the private sector of the economy. Lately, methodologies such as systems analysis, operations research, performance budgeting, etc. have been applied increasingly to service oriented organizations in both the governmental and private sectors of the economy. It is our belief that these methodologies can be employed to provide substantial benefits when applied to the management of university libraries.

There are those who feel that scientific management approaches can not possibly be applied to library decision making because of the complexities, intangibles, and uncertainties involved. We agree with these critics that decision making for libraries is a very difficult task. However, we point out that decisions are being made by decision makers according to some set of criteria. It is our belief that the development and employment of a management decision-making and planning framework will enhance the likelihood that effective decisions and plans will be made regarding the allocation and utilization of the scarce resources available to the library.

The development of a decision-making structure tends to formalize the decision making process. This formalization of the decision process provides the library administrator with an expanded awareness of the available alternatives and of the ramifications inherent in the selection of a particular alternative. In addition, new alternatives may be uncovered in this process. This increased awareness aids the administrator in making more rational decisions which are consistent with the overall objectives of the library. Decisions made in this fashion increase the overall effectivenss of the library in meeting the needs of its users. In addition, this formalization of the decision-making process provides a framework for the employment of techniques such as computer applications, mathematical models, and work measurement, which save time, aggregate and analyze information, enhance the precision of decisions, etc. Finally, a sounder basis is provided for justifying current expenditures and requests for additional funds through the employment of a management structure. This ability of a library to justify requests and expenditures of funds is gaining increased importance in negotiations with the university administration.

## Scientific Decision Making as Applied to University Libraries

To date there have been relatively few efforts to apply scientific management methods to the library decision-making process. The contributions that have been made in this area have occurred in the past few years. Probably the most substantial contributions in this field have been made by Philip Morse in Library Effectiveness: A Systems Approach[1], by Jeffrey Raffel and Robert Shishko in Systematic Analysis of University Libraries[2], and by M. K. Buckland, et al. in Systems Analysis of a University Library[3]. The work by Morse and Buckland, et al. represents experimental attempts to apply analytical models to various operating problems of the library. The work of Buckland, et al. is carried a little further than Morse's in the direction of actually implementing the results. Raffel and Shishko apply program budgeting techniques to the university library with a major emphasis placed upon cost accounting procedures. Their efforts are reported in the form of a case study as applied to the M.I.T. Library System. Other notable research relevant to decision making within the university library include Aridaman Jain's *A Statistical Study of Book Use*[4], Fussler and Simon's *Patterns in the Use of Books in Large Research Libraries*[5], Burkhalter's *Case Studies in Systems Analysis in a University Library*[6], and Roy, et al. *Operations Research and Systems Engineering Study of A University Library*[7]. None of these publications go very far in providing a comprehensive, integrated framework for use in the decision-making and planning process. There are a few other studies and models dealing with particular facets of library operations, as reported in the literature cited in the bibliography of this dissertation. In general, the publications in this field reflect the embryonic state of the art of the application scientific management methods to the problems confronting university libraries.

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[1] Philip M. Morse, Library Effectiveness: A Systems Approach, MIT Press, Cambridge, Mass., 1968.

[2] Jeffrey A. Raffel and Robert Shishko, Systematic Analysis of University Libraries: An Application of Cost-Benefit Analysis to the M.I.T. Libraries, MIT Press, Cambridge, Mass., 1969.

[3] M. K. Buckland, A. Hindle, A. G. Mackenzie, and I. Woodburn, System Analysis of A University Library, University of Lancaster Library, Lancaster, England, 1970.

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[4] Aridaman Kumar Jain, *A Statistical Study of Book Use*, Ph.D. thesis, Purdue University, Lafayette, Ind., 1968.

[5] H. Fussler and J. Simon, *Patterns in the Use of Books in Large Research Libraries*, University of Chicago Library, Chicago, 1961.

[6] Barton R. Burkhalter (ed.), *Case Studies in Systems Analysis in a University Library*, Scarecrow Press, Metuchen, N. J., 1968.

[7] R. H. Roy, et al., *Operations Research and Systems Engineering Study of a University Library, Progress Report*, Johns Hopkins University, Baltimore, Md., 1963 (PB 163 087).

Furthermore, developments in this area have not gained very wide acceptance, as demonstrated by the relative lack of actual implementation. There are some understandable reasons for this situation. Certainly the recency of these developments partially accounts for this slow rate of acceptance. Time is required to comprehend new ideas and techniques to implement new programs. The lack of qualified personnel with training in modern management concepts further impedes the implementation of these findings. Present library management personnel must be trained in these new concepts, or personnel with expertise in scientific management concepts must be recruited and oriented to the library process if these new findings are to be applied. An inability to comprehend how new management techniques and models fit into the overall decision and planning process further retards the adoption of these methods. Frequently library administrators have commented on the apparent impressiveness of various methods and techniques, but then proceed to express a lack of understanding as to how they would actually utilize the findings and results. This difficulty expressed by library administrators in making the transition from understanding the theory and operation of a new technique to actually applying and utilizing the findings and results of the technique in the decision process strengthens the need for an overall decision framework. This framework would provide the necessary guidance for incorporating these new concepts and results into the decision-making and planning process.

## Problems Requiring Decisions

Before proceeding further let us consider a sampling of some of the more important problems facing an administrator in the management of the library. These few problems give an indication of the types of problems confronting the administrator and provide insight into the need for a management framework for assisting in appropriate decision making.

1. Given that the library is allotted a specified amount of funds, how should these funds be allocated among the various collections? Within a collection, what is the optimum mix of titles, multiple copies, periodical subscriptions, etc. which should be purchased with the funds allocated to that collection?
2. If the amount of funds allotted to the library is changed by 10 per cent, how does this affect the level of service offered? Can this effect be justified to university budgetary officials?
3. What actions must be initiated to meet the changing needs of the university community as it changes in size and composition?

Additional problems are cited in Morse[8] and Haas[9], as well as other sources.

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[8] Philip Morse, Library Effectiveness: A Systems Approach, p.2.

[9] Warren Haas, "Research Library Management," Minutes of the 72nd Meeting of the Association of Research Libraries, pp.19-29.

Components of a Management System

Since there is an increasing library clientele which expects expanded and more rapid access to documents which are proliferating at an alarming rate, and since the available funds are increasing at a rate which tends to fall short of the rate of increase of costs of documents and services, it is essential that managerial decision making and the effectiveness of library service be improved. We believe that the development and employment of a comprehensive framework for decision making and planning bears the promise of a major contribution toward these goals of improved decision making and more effective service. The components considered vital for the success of this system are enumerated below.

1. A comprehensive yet succinct statement of the objectives of the library and related measures of effectiveness or performance must be delineated. Such a statement is necessary to provide guidance to the administrator for making plans and decisions which are consistent with the overall objectives of the library. This provides a basis for comparing the relative merits of alternative courses of action and for justifying expenditures and requests for funds.

2. A systematic method for enumerating and organizing feasible alternative courses of action is required. This process increases the administrator's span of awareness allowing him to consider and compare a large number of feasible alternatives. The probability of overlooking relevant feasible alternatives decreases when such a framework is utilized. In addition the structure should provide a means for relating service output with the associated resource input for each of the available alternatives.

3. An information system is required to insure that data and information relevant to particular problems are received by the manager responsible for the ensuing decisions. In addition to providing the information necessary for making decisions, this system should provide feedback on the outcomes of these decisions. This allows the manager to alter his decision mechanism and, where possible, to revise the original decision, if necessary.

In a climate of rapid change in the education process, it is important that this management system be sufficiently flexible and dynamic to adapt to and accommodate these changes. Demands for new and expanded library services or innovations in the manner in which services are offered should not necessitate structural changes in this management system.

Considering the current and projected economic conditions facing most university libraries in which resources for providing expanded service are limited, it is essential that library administrators use all available means to insure that these limited resources are allocated and utilized in the most effective manner. Recently a number of management tools and techniques such as benefit-cost analysis, computer techniques, analytical decision modelling, etc., have been developed. When properly employed these procedures can be of great assistance in arriving at decisions which substantially improve the effectiveness of the library. Hence, it is important that this management system be designed with the flexibility for incorporating and utilizing these tools and techniques. In particular, mathematical decision models, properly conceived and employed, can contribute significantly to the decision process. In the construction of a mathematical decision model for a problem, a detailed analysis of the relevant variables, including a determination of their relationship to each other and to the overall objective of the library is required. Performance of this analysis alone, tends to lead to improved decision making as a result of the increased awareness gained by the decision maker as he methodically considers the variables pertinent to the problem. This analysis necessarily precedes the actual development and solution of the analytical decision model. In addition, if the model is a valid conceptualization of the problem, solution of the model provides guidance in selecting the alternative which contributes most to advancing the objectives of the library. Finally, the variables determined to be relevant to the problem and its solution become the basic elements of the management information system. Procedures can be instituted to collect, synthesize, and disseminate the data and information descriptive of the variables relevant to the problem for use by the manager responsible for the decision.

## The Management System

In this dissertation a management system is developed which attempts to meet the criteria outlined in this chapter. The system is designed to be responsive to the changing needs of the university community as well as to be adaptive to innovations and advances in library technology. The system provides a comprehensive, rational framework to assist administrators in making decisions which are consistent with the objectives of the library. This framework provides the decision maker with a more expansive overview of the problems and alternatives confronting the library, thereby enhancing the likelihood that the best decisions are made. Finally, modern management techniques are easily integratable into this system.

The development of this system begins in Chapter II with an analysis of university library objectives. Criteria are set forth specifying certain attributes required of an objective if it is to be of value in the management process. On the basis of this and a survey of relevant research in this area, a succinct statement of objective and associated measure of performance is formulated for a university library. Utilizing the concepts of Planning-Programming-Budgeting, a decision framework is developed in Chapter III for organizing the decision process and for relating resource inputs to service outputs. A typical program structure specifying programs and subprograms for all library service offered and operations performed is outlined. In addition, criteria and associated decision rules utilizing benefit-cost analysis are formulated for the allocation of funds among and within the various programs specified in the program structure. In Chapter IV analytical decision models for some of the more major subprograms are constructed which relate resource inputs to service outputs in relationship with the library's overall measure of performance. In Chapter V these models are validated and associated parameters estimated. Finally, components of a management information system are developed in Chapter VI with special emphasis on specifying the variables for which data and information are collected, synthesized and disseminated for use in the decision-making process.

## A Caution

To this point we have focused primarily upon the benefits yielded by the employment of such a system. Not much has been said regarding the cost of employing and operating such a system. Obviously an organization cannot transform from "seat of the pants" to scientific decision making without making an investment. The reality of the situation is that the employment of any new system, especially one as pervasive and encompassing as the one developed in this research, requires a tremendous effort and involvement on the part of the library management staff. Up to a point where diminishing returns set in, the more effort devoted to implementing such a system, the greater the returns tend to be. In addition, the initial effort required to initiate and implement a new system is even greater than the effort required to operate and maintain the system in succeeding periods. The amount of effort required to accomplish this is often an overlooked point and the source of much dissatisfaction regarding the employment of any new technique. Too often managers expect phenomenal results from a new system without wanting to undertake the responsibilities necessary for its implementation and operation. After a minimal amount of effort is expended and miraculous results are not realized, the whole concept is discarded and old patterns are resumed. There is no management system yet known which provides exceedingly useful results for a small expenditure of time and effort. However, it has been demonstrated that given a concerted effort and commitment to the employment of scientific management methods, benefits are reaped in terms of improved plans and decisions resulting in better utilization of scarce resources and in providing sounder bases for justifying expenditures and requests for funds.

# CHAPTER II

# OBJECTIVES AND PERFORMANCE MEASURES

## Introduction

The effectiveness of any system depends in part upon an explicit set of objectives describing what the system is attempting (or should be attempting) to achieve. This set of objectives, in addition to being comprehensive, should be free of internal conflicts, or, means should be specified to resolve conflicts when progress toward one objective is detrimental or sacrifices progress toward another. Equally important is that this set of objectives permeate and become an integral part of the decision-making process at all managerial levels. This dictates that managers at all levels become aware of and understand the system's objectives and how each decision affects these objectives. Without a set of precise and consistent objectives which become an integral part of the decision-making process, administrators at various levels in the system cannot be expected to make decisions and formulate plans which yield the greatest contribution to what the system is attempting to achieve. The effectiveness of a university library is no exception to this tenet.

In conjunction with the development of a set of objectives for a system, it is extremely important to develop a corresponding set of quantitative performance measures. These performance measures can be utilized to evaluate the overall effectiveness of the system in attaining the stated objectives, as well as to aid managers in the decision process for evaluating the effects of alternative courses of action. Everyday decisions are made to allocate scarce resources among alternative programs within the library. If these resources are to be allocated so as to maximize the library's objectives, it is essential that means be available for evaluating the effect of alternative schemes of resource allocation in terms of their contribution to the library's objectives.

In this chapter an overall objective for a university library as a system is developed. This objective is addressed to the "raison d'etre" of the university library (or what the library is attempting to achieve) in terms of output of benefits. Since the university library is an integral sub-system of the university, the objective developed for the university library relates to the objectives of the university. In addition, a quantitative measure of performance corresponding to this objective is specified. Finally, the relationship of standards to objectives and performance measures is explored.

## The University

Before considering the objectives of a university library, we wish to consider the general concept of a university. There appears to be no universally accepted definition of a university. However, for the purpose of this research, the following statement provides an adequate definition of a university for our purposes.

> (placing) emphasis on scholarship and research... Its components are, typically, an undergraduate college and a cluster of graduate and professional schools. The graduate school stresses research training in an academic discipline; professional schools emphasize the development and application of theory in such fields as law, medicine, engineering and business.[1]

Ultimately, the objectives of the university are to enhance the attainment of society's objectives. These objectives can be stated in terms of the functions performed in the areas of instruction, research and community service. The university offers instruction to stimulate the educational growth of its students who then apply these abilities to further the objectives of society. Research is conducted in many areas and at many levels by faculty, staff, and advanced students to discover and apply new knowledge for enhancing societal objectives which mutually benefits the university, its members and society. In recent years some universities have been called upon to assist in meeting the needs of the neighboring community. To date the university's role in this area is rather minor and ill defined.

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[1] American Council on Education, American Universities and Colleges, 10th Edition, 1968, Washington, D. C., p.6.

For this reason this latter function will not be included as part of the objectives of the university in the present research.

Since we are developing a set of objectives general enough to apply to virtually any university library, a more specific set of university objectives is not necessary. The preceding description of what a university is attempting to achieve suffices for the purpose of this study.

## Library Objectives

The primary objective of a university library is to maximize future expected benefits imparted to society at a given cost of operation. Benefits to society occur when members of the university community in some manner apply what they have obtained from the university library for the betterment of society. The university community is defined to include students of the university, members of the faculty and professional, administrative and supporting staff members.

At this time, determination of future expected benefits imparted to society by members of the university community is an insurmountable task. A more feasible approach than attempting to analyze the interface activity between members of the university community and society is to focus on the interface activity between the library and members of the university community. In this regard it would be desirable to ascertain the benefits derived by a user of the library in terms of knowledge gained and/or abilities developed. Since measurement of these benefits, either quantitatively or qualitatively proves to be exceedingly difficult, we drop back one more step and focus on something that is measurable; the exposure of an individual to the library. Once an objective is specified in terms of exposure of individuals to the library, inferences can be made regarding value of knowledge gained or abilities developed by these individuals when applied to societal objectives. Figure 2.1 illustrates this concept.

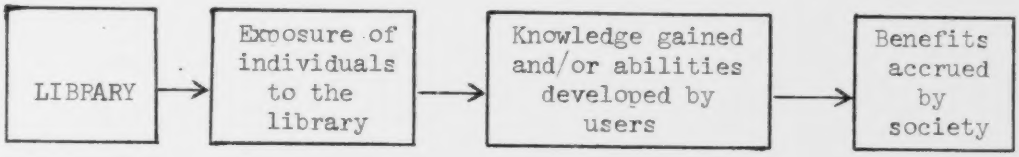
LIBRARY
Exposure of individuals to the library
Knowledge gained and/or abilities developed by users
Benefits accrued by society

Fig. 2.1--University Library Benefits

We must further consider that the university library is not an autonomous unit. Rather, it is a component of a larger system; the university. As the university library plays an important role in supporting university programs, objectives for the university library must reflect this role. In addition, the library is expected to be responsive to the needs and requirements of its users. In the long run, supporting university programs and serving the needs of members of the university community are not incompatible. It may well be that in the short run the library supports programs which generate little demand; for example, the development of a South East Asian collection prior to the advent of students and faculty in this area. However, when the objective is specified in terms of meeting future expected needs or demands, this apparent conflict is resolved. Hence, the objectives can be defined in terms of the needs and requirements generated by current and future personnel involved in the teaching and research efforts of a university.

The following statement adopted by the ACRL Board of Directors represents a basis for a workable objective:

> The primary purpose of any library is to serve the reading, reference and research needs of its users. All authorized users of college and research libraries have a right to expect library services up-to-date and commensurate with their needs, provided by competent librarians and founded on adequate collections which are easily available in suitable quarters.[2]

Unfortunately, enumeration of this objective, in terms of the collection, its accessibility, its availability, assistance in its use, instruction in its use, physical facilities, and open hours does not lend itself to meaningful quantitative evaluation in terms of performance measures. Examples of specific statements include:

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[2] "Statement of Service to Library Users", Statement approved by ACRL Board of Directors at Midwinter, 1966 meeting, ACRL News, No. 2, April 1966.

> The library collection...should be sufficiently broad, authoritative and up-to-date...;
>
> The collection should include...adequate bibliographic tools to exploit the collection efficiently;
>
> Help and advice in the use of the library...should be available from competent professional librarians.[3]

Objectives for a university library which are concerned with the benefits derived from the user-library interface, are related to the objectives of the university and have a potential for being quantitatively evaluated are presented in a report describing library research conducted at The Johns Hopkins University. The primary purpose of the university library is "to provide the patrons of the library with library service in the most efficient way."[4] In this report total cost is defined to include the cost of providing library service in addition to the cost suffered by patrons when a need is not met. The objective is then restated "to minimize the total cost of the library system to the university."[5] Here both the library and the patrons are considered as part of the library system. Since the cost of unsatisfied library needs is extremely difficult to measure, the objective is restated in terms of maximizing library service offered by the library per dollar cost of library operations. Library service is delineated in the report as follows:

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[3] Idem.

[4] "Progress Report on An Operations Research and Systems Engineering Study of A University Library", (NSF Grant NSF-GN-31), The Johns Hopkins University, Baltimore: April 1963, pp.27-33.

[5] Idem.

1. to provide and maintain as much as feasible of the library material needed by patrons in the research and teaching efforts of the university
2. to supply as much document identification information as feasible about library material needed by patrons
3. to minimize the time required for patrons to obtain and absorb needed information.

Unfortunately this statement of objective is not internally consistent. Inherent in this objective are statements which are in conflict from the viewpoint that achievement in one area necessitates sacrificing achievement in another. For example, providing as much material as feasible could very well consume the entire library's budget, leaving little or nothing for providing document identification information. No means are presented for resolving these conflicts.

Admittedly, providing material, providing document identification information, minimizing time to obtain and absorb information, along with providing study space, providing access to documents in other libraries, etc., are all valid objectives for a university library. However, increased effort applied in one area most certainly reduces effort applied in another.

One solution for resolving the conflicts arising when a system is striving toward multiple objectives involving tradeoffs or sacrifices of effort is the formulation of a higher level objective. Then, to the extent possible, the other objectives are related to the overall, higher level objective.[6] This approach is to be pursued in the present research.

The one objective that most librarians and users agree upon as the primary, overall, objective of the library is to bring together members of the university community and documents of recorded human experience. Members of the university community can be defined into population subsets along two dimensions. The first dimension is in terms of university affiliation: undergraduate student, graduate student, staff, and faculty. The second dimension is in terms of subject area of interest. For purposes of this classification, subject areas are defined as subdivisions of an academic discipline in which the subject matter or knowledge of any particular subject area is rather homogeneous. To the extent possible these subject areas are defined to be non-overlapping and to cover all areas of knowledge within a discipline. Here academic disciplines are referred to as subject categories. For example, physics is a subject category; whereas nuclear physics, high energy physics, solid state physics, gravitation, and astrophysics define subject areas within physics. In this research study subject areas were defined with the cooperation of professors in the respective academic disciplines and library personnel. The criteria for defining a subject area consisted of a combination of the way the discipline was viewed by the academic profession and what seemed to comprise a meaningful area for document selection purposes. Some consideration was also given to how these areas correspond to areas defined by the Dewey Decimal (D.D.) and the Library of Congress (L.C.) classification systems. A listing of tentative subject categories and areas developed at the University of Pennsylvania is given in Appendix A. Documents can be divided into document subsets according to form and subject matter. Document subsets defined by form include books, periodicals, microform, audio-visual material and non-book material not requiring special equipment for use, such as government documents, maps, manuscripts, etc. Document subsets defined by subject matter are identical to the subject areas used for specifying population subsets.

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[6] Russell L. Ackoff, A Concept of Corporate Planning Wiley-Interscience 1970, N.Y., N.Y. p.31.

With the defining of population subsets and document subsets a more formalized statement of the objective of a university library can be phrased. This is to maximize the expected future exposures of university community members to documents given current resources and anticipated future expenditures. Current resources include the assets actually owned by the library (e.g., documents, equipment and buildings), whereas anticipated future expenditures include those funds received from the university, private gifts, governmental agencies, and fines which are to be expended for various programs. The concept of exposure will be treated more rigorously in the next section.

## Performance Measures For Decision Making

Before a performance measure corresponding to the preceding objective can be developed, the exact nature of an exposure must be specified. Ideally we would like to measure the benefits accruing to society when individuals apply what they have gained from the library to society. In lieu of this we would like to determine the benefits gained by individuals as a result of utilizing the library. Since measurement of any of the foregoing is beyond the realm of this research, we might consider measuring the amount of time a patron is exposed to a document. Exposure time would accrue as a result of a patron consulting a document directly within the library or consulting a document which has been borrowed. Data collection requirements for computing this measure would include user surveys to determine how a user spends his time in the library; a survey to ascertain the amount of time that users consult documents which have been circulated; and a survey of service staff to determine the amount of their time spent consulting documents to answer users' requests.

An exposure measure requiring less data is the item-use-day originally proposed by Meier.[7] An item-use-day is defined as one document used during any portion of one day. Item-use-day data can be collected by adding the number of recorded charges (reserve desk, periodical desk, closed stacks, etc.) to the number of documents left out on tables and desks by users at the end of the day. To estimate the number of item-use-days experienced by a book which circulates requires the administration of a questionnaire to a sampling of users. We would expect that the relationship between the number of item-use-days per book circulation and the length of the loan period would be monotonically increasing as depicted in Figure 2.2.

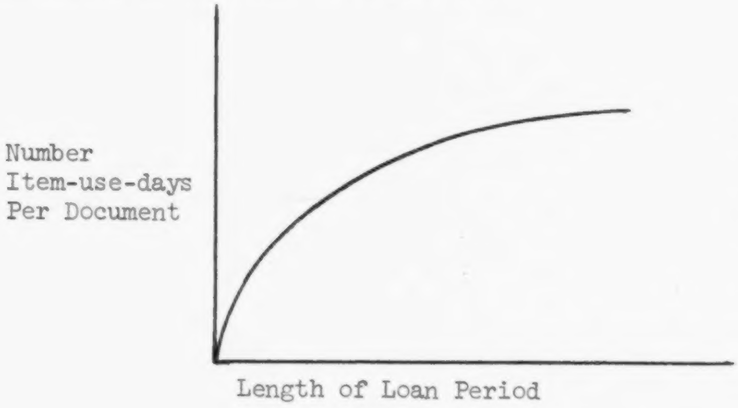

Fig. 2.2--The Number of Item-use-days as a Function of The Length of The Loan Period for Circulated Books.

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[7] R. L. Meier, "Efficiency Criteria for the Operation of Large Libraries," Library Quarterly, 31:215-234 (July 1961).

Except for ascertaining an estimate of the number of item-use-days per circulation, there is no need to administer a costly questionnaire to users of the library to compute this overall measure of exposure. However, the item-use-day measure does not seem to yield as reliable results as the exposure time measure in terms of inferred benefits. For example, three one-hour exposures to three different documents count three times as much as one three-hour exposure to a single document when utilizing the item-use-day measure. In general we would not expect the former type of exposure to be worth three times the latter type. However, although the item-use-day measure is not as reliable a measure as exposure time, the expediency involved in collecting the data required in the construction of the measure makes this a very desirable measure. Considering the savings of time and effort involved in obtaining exposure data for computing the number of item-use-days, this measure of exposure is used in this research.

Therefore, if we let

$E_i$ = the annual number of item-use-days for document subset i,

then given current resources and anticipated future expenditures, decisions can be made consistent with maximizing $\sum_i E_i$ over the useful life of the program.

If exposures to documents of certain document subsets are deemed more valuable than others, in terms of the inferred contribution to society's objectives, weighting factors can be employed to accommodate these differences.

If we let

$V_i$ = estimated relative average value of item-use-day exposures to documents in document subset i,

then the library might wish to maximize $\sum_i V_i E_i$. We can specify that $V_i = 0$ if a document is judged to possess no social value and $V_i = 1.0$ if a document is judged to possess the greatest possible social value. Other assignments can be made between zero and one depending on the relative social value of the document.

Administrators are prone to react negatively when asked to place a value on a document. We would like to point out that, if an administrator feels that exposure to certain subsets of documents is more valuable than exposure to other subsets, and considering that he makes decisions entailing funding allocations for documents, then these judgments of relative value are made in an informal manner. What we are proposing is that these judgments of relative value be delineated in a more formal manner.

Some library administrators might find it desirable to subdivide document subsets further according to whether the document is primarily used to support the research or teaching programs of the university. Classification of documents according to their primary purpose would yield three different sets: those documents which definitely support research; those documents which definitely support the teaching program; and those documents which support both. This concept is depicted in Figure 2.3.

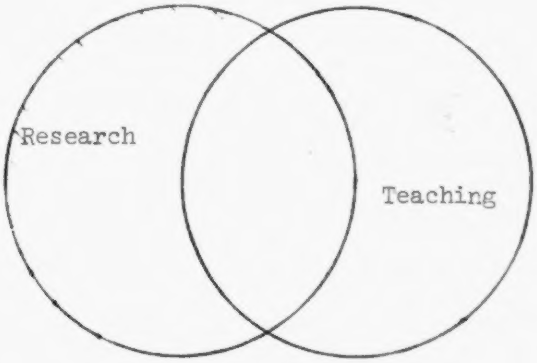

Fig. 2.3--Classification of Documents According to Support of Research and/or Teaching Programs of a University.

There is not a clear cut distinction between the research and teaching efforts of a university. For example, how should the efforts of a student, who is preparing a research paper for a graduate seminar, be classified? Likewise, how should the documents used to support such efforts be classified? To avoid becoming enmeshed in a philosophical discussion of this point, a set of criteria developed at the Columbia University Library designed to assist in making this distinction is presented.

RESEARCH - Primarily used for:

1. Faculty and research personnel's research projects, whether funded by local or personal means or by outside agencies.

2. Keeping faculty members and other research personnel up to date in their fields of research.
3. Preparation by students of doctoral dissertations, master's essays and major research papers in law or medicine.

INSTRUCTIONAL - Primarily used for:

1. Students' work in connection with courses, whether undergraduate or graduate.
2. Faculty members' preparation for classroom presentation or general background development useful in the classroom, or guidance of the work of graduate students toward advanced degrees.

BOTH RESEARCH AND INSTRUCTION - Used about equally for Research and Instruction.[8]

To ascertain the relative percentage of document exposures which support the research and teaching activities of the university, a sample of documents consulted in the library and circulated could be evaluated. The documents would be classified according to the three categories: research only; teaching only; and research and teaching. Those documents classified as both research and teaching would be apportioned to the research only and teaching only categories in accordance with the relative number of documents in each of the two classes in order to eliminate the combined research and teaching class.

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[8] "Cost Analysis of the Columbia University Libraries For the Fiscal Year 1968-69", Unpublished report, Columbia University Libraries, June 1970.

Therefore, if the research/teaching distinction is desired, the performance measure to be maximized over the future years can be represented as:

$$\sum_i \sum_k V_{ik} E_{ik} \qquad (2.1)$$

where $E_{ik}$ = annual number of item-use-days of documents in document subset i which support university program k, where k = 1, 2 correspond to research and teaching programs respectively,

amd $V_{ik}$ = estimated relative average value of item-use-days for documents in subset i which support university program k, where k = 1, 2 correspond to research and teaching program respectively,

given current resources and anticipated future expenditures.

The next step in the development of the performance measure is to incorporate the concept of maximizing benefits over future years or a planning horizon. To accomplish this we redefine the item-use-day variable to include a time dimension. For this we let

$E_{ikn}$ = the expected number of item-use days of documents in subset area i supporting program k during year n.

Then the measure to be maximized becomes

$$\sum_n \sum_i \sum_k V_{ikn} E_{ikn} \qquad (2.2)$$

given current resources and anticipated future expenditures.

It might be argued that future benefits should be discounted. That is to say, some might view an immediate exposure to be more valuable than an exposure in n years. One reason for this logic might be termed the "acceleration effect of benefits." Often one benefit will precipitate another which in turn will precipitate another, etc. The more quickly benefits occur, the closer we come to attaining the objectives of society. From this standpoint, the more quickly an exposure occurs, the more value it possesses. The concept of discounting future benefits gains credence from an economic standpoint if these benefits were expressed in monetary terms. When benefits are expressed in monetary units, discounting to obtain present values is considered a normal function in the decision process. To express benefits imparted by the library in monetary units requires that $V_{ik}$ be defined in terms of the dollar value of an item-use-day. Hence it can be argued that if benefits expressed in dollars would be discounted, then benefits expressed in any other units should receive similar treatment.

To accommodate this we let

$r$ = annual rate at which future benefits are to be discounted,

then the present worth of the weighted item-use-days which are expected to occur in year n for exposures in subject area i, supporting program k, is

$$\frac{V_{ikn} E_{ikn}}{(1 + r)^n} \tag{2.3}$$

The performance measure to be maximized becomes

$$\sum_n \sum_k \sum_i \frac{V_{ikn} E_{ikn}}{(1 + r)^n} \tag{2.4}$$

given current resources and anticipated future expenditures.

To determine estimates of the value of r, questions of the following type could be posed to library administrators, advisory committee members, university officials, etc. How does the value of an exposure this year compare with the value of an exposure in five years? An iterative technique, which asks a person to indicate his preference between two alternatives, such as 90 exposures this year versus 100 exposures in 5 years, could be employed to assist in this process. When the person indicates that he is indifferent between these two alternatives, the following expression can be solved for r.

$$F = (1 + r)^n P \tag{2.5}$$

where P = number of exposures in current year

F = number of exposures person indicates to be equivalent to P in n years

n = time frame in years

Alternatively a case can be made for simply using a discount rate of money. In many situations benefits are yielded in or can be converted into monetary units. In these instances the difference between the monetary benefits and monetary imputs is discounted over time using the discount rate of money. It can then be argued that if monetary benefits are discounted at the rate r, then other benefits should be discounted at a similar rate. The discount rate of money would most assuredly be used for r if $V_{ikn}$ were valued in monetary terms. In any case we would expect that reasonable values of r range from 0 to .30.

Finally, in order to evaluate the desirability of alternative courses of action, it is important to incorporate a measure of the cost of current resources and the anticipated future expenditures into the performance measure. In this manner the merits of implementing various programs can be evaluated by comparing the expected benefits to cost ratio associated with each program. In this connection we are concerned with two types of funds: the value of current resources and the stream of future expenditures. We treat the value of current resources first.

Current resources of the library include such assets as building space, book collections, equipment, etc. These have monetary value only to the extent of the money which could be derived if these resources were rented or sold. In this respect we are concerned with the current market value of the resources rather than with past expenditures. Thus the monetary value of the library's current resources equals the dollar amount obtained if these resources were put to alternative use.

This concept of attaching a monetary value to the current library resources and including it in the performance measure is valid only if the library administrator is open to the idea of liquidating certain assets. Examples of decisions in this category might include renting out of part of an under-utilized building, sale of a collection because the associated academic department has been discontinued, etc. If consideration of these types of decisions is forbidden (i.e., if the library never considers the idea of selling or dispensing with anything), then the monetary value of current resources need not be considered as part of the performance measure when evaluating various programs and courses of action. All assets are then considered as sunk costs. This is not to imply that rational decisions can be made without regard to resources available. The extent of current resources must be considered in any decision whether or not a monetary value is attached to these resources.

The other monetary consideration affecting the performance measure concerns the stream of funds to be expended in the future for each program and for the entire library. The term future expenditures includes what is sometimes termed present year expenditures as well as future year expenditures. It is meant to include all expenditures from this point in time onward. To analyze these cost considerations, we focus on the time relationship between expenditures and the benefits resulting in the future. Specifically three situations are contrasted: an immediate expenditure designed to yield an immediate benefit; and immediate expenditure designed to yield an equal benefit in t years; and an expenditure in t years designed to yield an equal benefit in that year. This is depicted diagrammatically as follows.

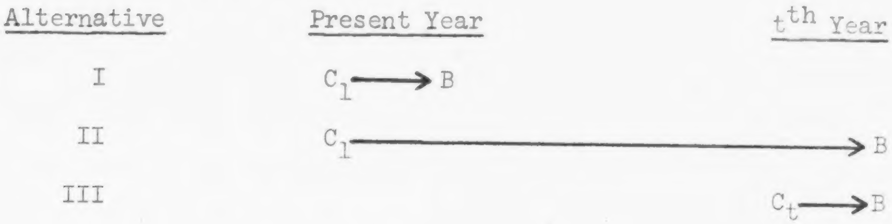

Fig. 2.4--Cost-Benefit Timeframe

In this diagram Alternative I considers the cost of an immediate expenditure, $C_1$, to yield an immediate benefit, B. Alternative II considers the cost of an immediate expenditure, $C_1$, to yield an equal benefit, B, in t years. Finally, Alternative III considers the cost of an expenditure in the $t^{th}$ year, $C_t$, to yield an equal benefit, B, in that year.

Considering only the discount rate of money, $C_1$ would be larger than $C_t$, when $C_t$ is discounted back to the present year. Therefore, given that

$i$ = annual discount rate for money,

then the value of $C_t$ at the present time is $\dfrac{C_1}{(1 + i)^t}$.

On the other hand considering only the effect of inflation, $C_t$ would normally be larger than $C_1$. That is to say, the cost in the $t^{th}$ year to produce one unit of benefit in the $t^{th}$ year is greater than the cost in the current year to produce an equal unit of benefit during the current year. Therefore, if we let

$q$ = annual rate of inflation for library related expenditures

then $C_t$ equals $(1 + q)^t C_1$, which is greater than $C_1$. Hence, if we budget for n years in advance, based upon current year costs, in general we would expect the actual costs in the $t^{th}$ year to be $(1 + q)^t$ times as great as the budgeted costs.

Therefore, considering the factors which arise because of the time dimension, we will proceed to compute the benefit-cost ratios at the present time for each of the three alternatives.

| Alternative | Present Time Benefit-Cost Ratio | Time Factors Present |
|---|---|---|
| I | $\frac{B}{C_1}$ | None |
| II | $\frac{B/(1 + r)^t}{C_1}$ | Benefit Discounting |
| III | $\frac{B/(1 + r)^t}{C_1(1 + q)^t/(1 + i)^t}$ | Benefit Discounting,<br>Monetary Inflation, and<br>Monetary Discounting |

Figure 2.5--Cost-Benefit Ratios for Differing Timeframes

Finally, we generalize these concepts to compute a performance measure for a program which is expected to yield item-use-days in document subset i in support of university program k distributed over a number of years for a cost distributed over a number of years. In terms of the performance measure of the future expected weighted item-use-days given current resources and a stream of future expenditures, we have:

$$\frac{\sum_n V_{ikn}E_{ikn}/(1 + r)^n}{C_{iko} + \sum_n C_{ikn}(1 + q)^n/(1 + i)^n} \tag{2.6}$$

where $C_{iko}$ = present market value of current resources associated with a particular program in subject area i supporting university program k

and $C_{ikn}$ = expected expenditures in year n associated with a particular program in subject area i supporting university program k

This measure of expected weighted item-use-days per dollar is instrumental in evaluating the desirability of a particular program when determining where funds should be appropriated. If a program can be funded at different levels to yield different levels of benefits, the marginal value of this ratio becomes the significant factor in determining the optimal funding level so that the library operates at maximum effectiveness. The use of this marginal benefit per unit cost ratio in the decision process is discussed in greater depth in Chapter III.

At this point we would like to emphasize that the objective of maximizing expected weighted item-use-days per dollar appears to be the primary but not exclusive objective of a university library. Obviously, there are multiple objectives such as providing study space, performing an archival function for certain documents, minimizing document retrieval time, providing reference service, maintaining certain complete collections, etc. Up to a point, progress toward one of these "secondary objectives" is consistent with the "primary objective" of maximizing exposures. For example, the shorter the retrieval time of documents, the greater the document exposure. The more study space provided, the greater the document exposure. In most cases inferences can be made relating progress toward the attainment of these "secondary objectives" with the effect upon the item-use-day per dollar measure. Given these relationships it is then possible to determine the optimal amount of resources to devote to the attainment of the "secondary objectives" such that overall exposures are maximized. In other cases certain objectives might be treated as constraints. For example, a library might be required to purchase all documents within a specific subject area so as to maintain a complete collection as part of an inter-library network agreement. In the next chapter we discuss in more depth how some of the more secondary objectives can be handled.

## Standards

In general a state of confusion seems to exist on the part of library administrators as to the difference between standards and objectives. The term "standards" as used by the library profession seems most analogous to "goals" as used by managers in the private business sector. In the private business sector a goal is defined as a level or point on a performance measure scale which a system is attempting to achieve within a specified period of time (see Ackoff).[9]

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[9] Ackoff, A Concept of Corporate Planning, p.31.

For example, one objective of a university library might be to minimize the average document retrieval time for documents borrowed through an inter-library loan system. A goal in this area might be to decrease the average time required to obtain a document via inter-library loan to 3 days over the next two year period. Hence, a goal represents a specific tangible state or point which the system is attempting to attain within a specific timeframe.

For the most part, even the standards which are recognized as goals and not objectives possess many shortcomings. Most current university library standards fail to provide a basis for meaningful quantitative evaluation of the performance of the library for the following reasons.

1. Most standards are descriptive in nature making evaluation extremely difficult.
2. Those standards which prescribe quantitative goals were formulated by value judgements which by their nature are of questionable value.
3. The primary emphasis of standards is directed toward setting goals for library resources (a static concept) as opposed to setting goals for the output or benefits imparted in the user-library interface (a dynamic concept).
4. Most published standards are insensitive to the particular user needs and requirements which are generated as a result of involvement in programs which differ in content and intensity from university to university.

The recently published data for 50 of the better university libraries in the U. S. and Canada by the Committee on University Library Standards[10] meet the criteria of being quantitative and not dependent upon value judgements. However, the major emphasis is still directed toward measuring inputs to the library or resources as opposed to measuring outputs or benefits imparted to the users. The one exception is in the area of reserve and circulation service wherein data are presented on a per student basis as well as on a library basis. In addition, the defining of input measures in the areas of expenditures, document resources, personnel and space on a per student basis represents a significant improvement in the development of input indicators (e.g., volumes per student). Development of comparable measures relating to the research function of the university would represent a further significant advancement in the area of input indicators.

Unfortunately these standards are not very helpful in evaluating the effectiveness of any particular library. They merely serve as a guide for comparing gross input data in the areas of expenditures, space, personnel and document resources and circulation data of a particular library with gross input data and circulation data for 50 of the better university libraries in the U. S. and Canada.

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[10] Robert Downs and John Huessman, "Standards for University Libraries", College and Research Libraries, 31:28-35 (January 1970).

A contribution toward the establishment of standards for academic library collections which could be quantitatively evaluated in terms of the instruction and research programs of the academic institution was made by Clapp and Jordan. They were interested in:

> producing estimates of the size required for minimum adequacy by the library collections of a number of academic institutions of widely differing characteristics. It was important that these estimates should carry conviction to the planning, budgeting, and appropriating bodies concerned. Available standards were found unsuitable for producing the desired result. Accordingly, an attempt was made to develop formulas in which separate account would be taken of the principal factors that affect the requirements for books in connection with academic programs, and in which each factor would be weighted in a manner capable of being related to and justified by practice.[11]

Although this approach attempts to specify collection objectives which are quantifiable and which relate to the teaching and research programs of the institution, they are nonetheless prescriptive in nature; for example, 50 books per faculty member, based on a value judgment. Evaluation of the performance of a library in terms of benefits imparted to users is infeasible using these formulae; for example, how well user needs of a particular institution are met by the library collection at that university.

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[11] Verner Clapp and Robert Jordan, "Quantitative Criteria for Adequacy of Academic Library Collections," College and Research Libraries, 26:371-380 (January 1966).

Probably the greatest benefit derived from the use of prescribed standards on a national level has been the coercive effect felt by university administrators when they realized that their library did not meet current standards. Similar results may be expected when median measures of current library resources of a group of libraries are adopted as standards. After all, using these measures one-half of the libraries within this group will find themselves to be substandard!

## Performance Measure for Evaluating Library Effectiveness

In an earlier section we indicated that the measure of benefits to cost is very useful in the decision-making process. This measure provides a basis for assisting the administrator in determining how funds should be allocated among various programs to achieve maximum overall library effectiveness, i.e., maximization of the future weighted item-use-days. However, if comparisons are desired for the same library from one year to the next or between two different libraries, this measure is not a reliable indicator of a library's overall effectiveness. There are at least two reasons for the failure of this measure in yielding reliable results. One reason is that this measure is not adaptive to varying sizes and compositions of the university community. We would expect that the number of exposures would be a function of the size and composition of the university community. Given constant expenditures from one year to the next and an increase in the number of persons in the university community, we would expect an increase in the number of exposures per dollar. This increase would not necessarily reflect a change in the performance of the library. Hence if valid comparisons are to be made from year to year for the same library as membership changes or between two different libraries serving memberships which differ in size and composition, the performance measure must be normalized for these variables. Therefore, if we let

$P_j$ = the number of full time equivalent (f.t.e.) persons in population subset j

$W_j$ = the weight assigned to population subset j to reflect the relative number of item-use-days per member of the population subset j

then we would anticipate that a valid performance measure would need to contain the factor $\sum_j W_j P_j$, where j is summed over all population subsets. This factor incorporates a variable descriptive of size, as well as a variable descriptive of the composition of the university community.

Another reason for not using the benefit-to-cost ratio as an indicator of overall library effectiveness results from an understanding of what this ratio is actually measuring. In general we expect that within the range in which most libraries operate, the number of weighted item-use-days is an increasing monotonic function, at an decreasing rate, with respect to expenditures as depicted in Figure 2.6.

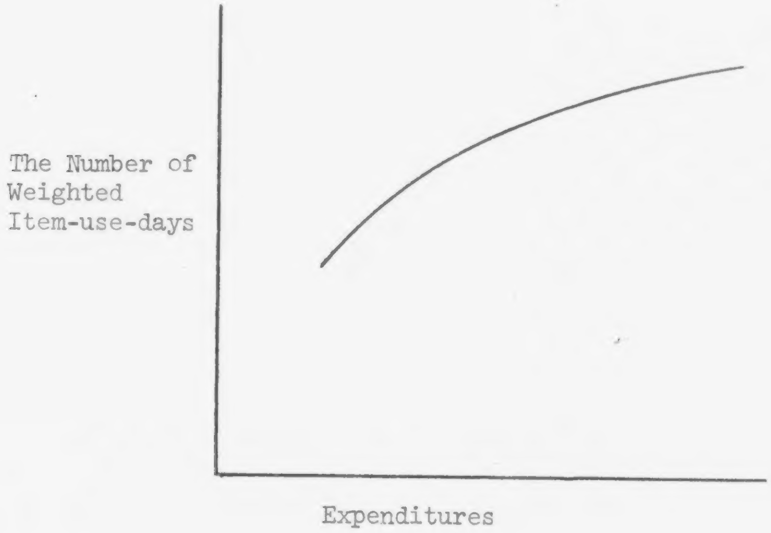

Fig.2.6--The Relationship Between the Number of Weighted Item-use-days and Expenditures.

Given the form of this relationship, we observe that, as expenditures increase the ratio of weighted item-use-days to expenditures decreases, even if effectiveness remains constant! We conclude then, that this measure alone is not so much an indication of effectiveness but rather an indication of the point on the curve at which the library is operating. Further, for a given level of expenditures and a constant membership, a change in library effectiveness would result in a shift in the position of the curve in Figure 2.6.

The only instance in which this measure would yield a valid indication of the effectiveness of a library from one year to the next is when the membership size and composition and level of expenditures remain constant. What is desired then, is a measure which is normalized for differences in size and composition of the university community and which is normalized with respect to the level of expenditure of the library.

In developing the performance measure for use in the decision process the orientation was in terms of the future. A measure for evaluating performance should be constructed to reflect the transactions or activity which occurs during a specific period of time. Hence, the measure should be constructed to include a count of the number of weighted item-use-days during the period as related to the dollars expended during this same period. The dollars expended should include the actual expenditures during this period in addition to a measure of the change in value of material library resources during the period. In view of the fact that there are seasonal influences affecting library exposure, we select one year as the time period covered by this measure. Therefore, if we let

$C_o$ = the market value of library resources at the beginning of the year

$C_o'$ = the market value of library resources at the end of the year

$C_1$ = total expenditures during the year

then $C_o - C'_o + C_1$ would comprise a measure of the expenditures which should be related with the benefits yielded during the period.

Efforts to develop a single measure which include variables for exposures, expenditures, and persons and which is a valid measure of performance have been unsuccessful to date. Before proceeding further, let us investigate what is meant by the term "valid." It would seem reasonable that a measure of the ratio of exposures per unit cost is a reliable indicator of one aspect of a library's effectiveness. The ratio of exposures per capita is a reliable indicator of another aspect of a library's effectiveness. Hence if a measure is to be a valid indicator of a library's overall effectiveness, it should include these concepts and provide a sound basis for relating these concepts within the measure. The problem evolves in determining the basis for the relationship between these two ratios. For example, how should an increase in the exposure per capita ratio, accompanied by a decrease in the exposure per cost ratio, be measured in terms of overall effectiveness? Before proceeding further, let us define the ratio of exposures per capita by the symbol $^E/_P$, where

$$\frac{E}{P} = \frac{\sum_i \sum_k V_{ik} E_{ik}}{\sum_j W_j P_j} \qquad (2.7)$$

and the ratio of exposures per expenditures by the symbol $^E/_C$, where

$$\frac{E}{C} = \frac{\sum_i \sum_k V_{ik} E_{ik}}{C_0 - C_0' + C_1} \qquad (2.8)$$

Then, the need for determining an equitable value for the tradeoff between the ratios of $^E/C$ and $^E/p$ can be seen more clearly in Figure 2.7.

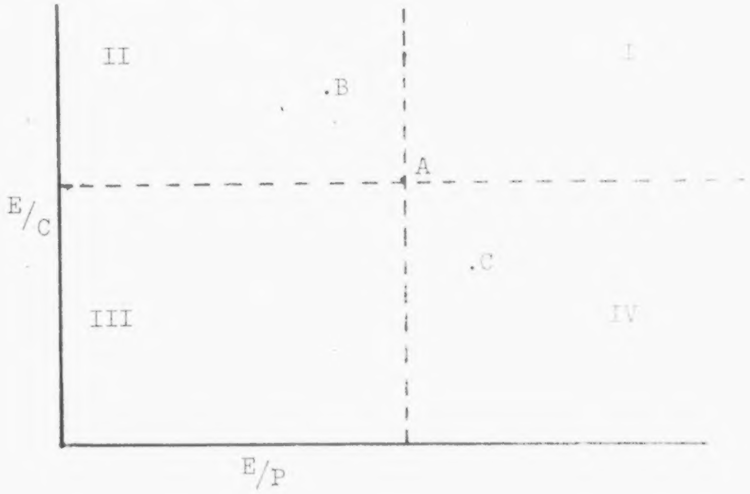

Fig. 2.7--Relationship Between Exposures per Expenditures, $^E/C$, and Exposures per Capita, $^E/P$.

In this figure it can be seen that any library in quadrant I is operating at a higher level of effectiveness than the library operating at point A. In this case any library in quadrant I has both a higher E/C and a higher E/P ratio than the library at A. In the same manner any library in quadrant III is operating at a lower level of effectiveness than a library operating at point A. In this case any library in quadrant III has both a lower E/C ratio and a lower E/P ratio than a library at A. On the other hand, a library operating at point B, any point in quadrant II, has a higher E/C ratio but a lower E/P ratio than the library at A. What does this imply about the overall effectivness of the library at B, relative to the library operating at A? Similarly a library at point C, any point in quadrant IV, has a higher E/P ratio but a lower E/C ratio as compared to the library at point A. What does this imply about the overall effectiveness of the library at C, as compared to the library at A? Finally, how does the effectiveness of the library operating at point C compare to the effectiveness of the library operating at point B? To provide a valid answer to these questions, an equitable relationship must be determined between these ratios. This would provide a basis for the partitioning of quadrants II and IV into "more effective" and "less effective" regions. This sort of partitioning is implicit in the development of a single indicator for measuring the overall effectiveness of a library. For example, a seemingly sensible single indicator of performance involves a measure of exposures per capita per expenditure.

$$\frac{E/P}{C} = \frac{\sum_i \sum_k V_{ik} E_{ik} / \sum_j W_j P_j}{C_o - C_o' + C_1}$$

This ratio yields consistent results in quadrants I and III of Figure 2.7, however partitions quadrants II and IV according to the relative magnitudes of P and C. This we believe is a rather arbitrary manner for determining the relative effectiveness of libraries in these regions.

Therefore, it would appear that if single measure of overall effectiveness is desired for evaluating a library's effectiveness from year to year or for comparing the effectiveness of two different libraries, some rather in depth analysis must be performed to arrive at an equitable relationship between $E/C$ and $E/P$. The term "equitable" in this sense refers to society's value for the relationship and trade-off between $E/C$ and $E/P$. Unfortunately a determination of this relationship is beyond the scope of this research. This represents a fruitful area for further investigation.

Problems associated with the use of any measure for evaluating library performance on a year-to-year basis are analogous to problems encountered in the private sector of the economy when profit is used. For example, a private corporation can show large profits in any given year by eliminating its research and development program. However, this is bound to affect corporate profits in a negative manner in future years. Similar situations are encountered in the university library when collections are built up prior to the formation of the departments which will utilize these collections. To accommodate this type of situation, procedures must be incorporated to match inputs with outputs during the period in which they occur. In this example, it would be necessary to defer the expenses associated with the development of a new collection until the department is formed and use is being made of the collection.

In addition, adjustments in computing this measure should be made to compensate for increases in the cost of library materials and services from year to year. To accomplish this, a price index might be constructed so that comparisons can be made which are free of inflationary influences.

It would seem apparent that evaluation of a library's performance from year to year or with other libraries is no easy task. However, we wish to reemphasize that although there remain unresolved problems in the area of evaluating past performance, and in making interlibrary comparisons, this does not affect the usefulness of the expected future exposures per dollar measure in the decision-making and planning process.

## Summary

In this chapter an objective was developed describing what a library is attempting to perform consistent with a set of criteria. Included in the set of criteria are requirements that the objective describe the primary purpose of the library, be free of internal conflicts, be clearly and explicitly defined for managerial decision making and planning, be consistent with the objectives of the university, and finally be quantifiable in terms of a performance measure. An investigation of the hierarchy of objectives pertinent to a university library began by exploring the purpose of the library in the broadest sense in terms of its ultimate contribution to the objectives of society. Next, we focused on the knowledge or abilities gained by users of the library. Since it is infeasible to adapt these concepts to meet the desired criteria, we next focused upon the exposure of users to documents of human recorded experience as the primary objective of a university library. In this regard the objective statement developed is *to maximize the expected future exposures of university community members to documents given current resources and anticipated future expenditures*.

In developing a measure of performance, item-use-day was selected as a measure of exposure in preference to exposure time. Data on the latter measure were considered to be too dificult to collect. A performance measure was developed which provides an indicator of the expected number of future weighted item-use-days per dollar of current resources and future expenditures. This measure and the associated measure of marginal exposures per unit cost prove to be extremely useful in the decision-making process.

The concept of standards was discussed with particular emphasis on an attempt to clarify the confusion which exists between standards and objectives. In addition some standards were evaluated in terms of their usefulness in the management process.

Finally, there was a discussion of the problems involved in constructing a single performance measure to evaluate library performance from year to year and to make interlibrary comparisons.

## CHAPTER III

## A LIBRARY DECISION-MAKING FRAMEWORK

### Introduction

In the preceding chapter a statement of objective was developed which focused upon the primary purpose served by a university library. In addition a measure of performance was developed to guide administrators in making decisions and plans consistent with maximizing overall library performance. At this juncture the need for developing a rational framework for decision making and planning is recognized. This framework is needed to assist library administrators in organizing and managing library operations and in allocating and controlling scarce resources among the various library activities. The Planning-Programming-Budgeting System (PPBS) was selected as a rational management framework to assist administrators in dealing with these tasks.

In this chapter a brief description of the components and attributes of PPBS is presented. Four dimensions are defined. (1) population, (2) documents, (3) functions, and (4) organization, as ways of classifying library activities into subsets. Utilizing this concept of dimensions, a typical program structure delineating programs and subprograms is specified. Finally, the concept of benefit-cost analysis is discussed with particular emphasis placed upon the use of benefit-cost measures in providing guidance for library administrators in determining an effective allocation scheme for funds among the various programs.

## The Planning-Programming-Budgeting System

A major problem confronting every library administrator concerns the manner in which decisions are to be made for allocating and controlling the library's scarce resources. Closely allied to this is the further problem of how library operations are to be organized and managed. The decisions and plans made in each of these areas have a direct effect upon the performance of the library in imparting benefits to society. If decisions and plans are to be made at all levels consistent with the overall library objective, it is important to employ a framework which assists in evaluating alternatives and which relates these alternatives to the overall objective of the library PPBS is one type of management system designed to achieve these ends.

PPBS represents a managerial structure useful for coordinating the decision-making and planning process consistent with the desired objectives of the system. Its major emphasis as a management tool is directed toward outputs or accomplishments as related to inputs or resources necessary to achieve these objectives. The essential components of PPBS include:

1. specification of the system's objective

2. derivation of performance measures to evaluate achievement of the objective

3. specification of a program structure oriented toward achievement of the system's objective

4. specification of input and output indicators for programs in the program structure

5. ascertaining the relationship between the output of a program and performance measures

6. development of a multi-year plan.

Attributes and advantages of utilizing PPBS as a management tool include:

1. emphasis on achievement of objectives

2. facilitation of consideration and evaluation of a wide range of alternative programs

3. assistance in optimal resource allocation

4. flexibility in adapting to idiosyncrasies of different organizations

5. compatibility with modern management methods such as benefit-cost analysis and mathematical models

6. provision of a rational and systematic means for decision-making and planning necessary for the functioning of any management system.

Further background discussion on PPBS applicable to education and

and libraries can be found in Hartley[1] and Raffel and Shishko[2].

Probably the greatest criticism leveled at PPBS is that it is difficult to implement. The reality of the situation is that establishing objectives, deriving performance measures, developing a program structure, specifying input and output measures and developing a multi-year plan does require a great deal of time and effort. If a library desires to know what outputs or benefits are being achieved per unit input to determine how resources can be most effectively allocated, and to justify current expenditures and requests for additional funds, it must be willing to undertake this process or a similar one.

Another basis for criticism results from the manner in which PPBS is implemented. Often PPBS is imposed upon a lower level agency from above. Hence frequently those involved with developing a PPBS believe that they are doing it to meet a requirement, without realizing what a helpful management technique it can be to them.

Before proceeding further, we define certain PPBS terms which are used in this study. These terms are adapted from Hartley[3].

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[1] Harry Hartley, Educational Planning-Programming-Budgeting, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1968.

[2] Raffel and Shishko, Systematic Analysis of University Libraries.

[3] Hartley, Educational Planning-Programming-Budgeting, p.167.

Program Structure: organization of activities into general program areas, sub-programs, elements, components, and tasks; it facilitates analytic comparisons of the costs and effectiveness of alternative programs; programs may cut across existing departments and agencies.

Programs are subdivided for managerial convenience in distinguishing between various parts of programs. Some programs may be subdivided in great detail, while others may be subdivided very little. The terms specifying the various sub-divisions are merely for convenience and there is no implication, for instance, that all components of various programs are of equal importance or that each program is of equal importance.

Benefit-Cost Analysis: a means of relating cost of a particular activity to the benefits imparted by that activity over a period of time; the decision maker may choose from among feasible alternatives on a basis of benefits imparted for a given cost.

Crosswalk: the expression of the relationship between the program structure and the appropriation-budget structure; the translation of multiyear program and financial plans into annual budgets; a simple table vertically listing program categories and horizontally listing appropriations and budget activites.

Project: a set of new activities which are non-routine and non-continuing, which have a specific beginning and closing date, which are outside the formal organizational structure, which generally relate to a single explicit objective, which normally relate to change and innovation and which cut across different programs.

## Dimensions

As an aid in viewing the activity of a library and in constructing a program structure, we specify certain relevant dimensions. Each dimension represents a way in which all library activity can be meaningfully classified into virtually non-overlapping and collectively exhaustive subsets. These subsets within a particular dimension are determined by subdividing the dimensions. Dimensions relevant to a library include: (1) population, (2) documents, (3) library functions, and (4) library organization. We considered the dimensions of population and documents in Chapter II. There, persons were classified according to their status at the university and subject area of interest, and documents were classified by type and form of the publication and subject area. What remains is a discussion of the dimensions for organization and functions. In this study the term "organization" refers to the manner in which the library activities are administratively grouped. The subsets of the organization dimension, termed library units, differ from library to library. However, in a typical university library, the library units are composed of the departmental libraries and the technical processing operations. Other library units might be specified to include certain administrative support units such as data processing, payroll and accounting, etc.

The subsets of the functional dimension are specified according to a combination of services offered to users and operations performed which are necessary for internal library use. As in the case of the other dimensions, subsets are so defined that they do not overlap an that they cover all library activities. Hence, virtually every activity a library performs can be classified under one and only one function. In addition, it is possible to identify and isolate the inputs and outputs of each function. A listing of these 18 functions follows:

Providing Physical Facilities

1. Provision of building area
2. Provision of user furnishings
3. Maintenance of facilities

Providing Access to Documents Within the Library

4. Selection of documents
5. Acquisition of documents
6. Classification of documents
7. Processing of documents
8. Control of document location and use
9. Facilitation of document use
10. Maintenance and weeding of documents

Providing Access to Documents in Other Libraries

11. Access to documents in other libraries

Providing Aids to Identify and Locate Documents and Conveying Information

12. Provision of aids to locate documents within our library
13. Provision of aids either to identify existing documents or to locate them in other libraries
14. Personal assistance for document identification and location and for conveying information

Promoting Library Use

15. Publications, advertisements, and exhibits
16. Personal communication with members of population being served

Planning, Administration, and Support

17. Planning and administration
18. Support activities

We have grouped these functions into six major areas: providing physical facilities; providing access to documents within the library; providing access to documents in other libraries; providing aids to identify and locate documents and conveying information; promoting of library use; and planning, administration, and support. These major areas evolved by grouping the functions according to similarity of the output of each function.

Some dimensions can be subdivided in more than one way. For example, for the university library, population can be subdivided according to status (that is, faculty, staff, graduate student, or undergraduate student) or by subject interests (e.g., anthropology, marketing, chemistry, mechanical engineering, etc.), or by both status and subject matter simultaneously (e.g., faculty-anthropology, graduate students-physics, etc.). Likewise, documents can be subdivided according to subject matter, by document type, by document form, etc. or by all simultaneously. If we consider that the smallest unit in any program structure is defined by subdivisions of the four dimensions, we realize there are an extremely large number of these units. For example, one unit might be defined as selecting (function) psychology periodicals (document) for undergraduates (population) for the Psychology Departmental Library (organization). Since a program can be defined as composed of a set of these units, it is apparent that there exists a tremendous number of possible program structures.

## Program Structure

One possible means of defining a program structure for a university library is according to subdivisions of a single dimension. Hence, programs can be defined by population subsets, by document subsets, by library functions, by library units or any combination of these. For example, a program structure defined by population subset might be directed toward meeting the needs of undergraduates. A program for a university library, utilizing a program structure defined by documents, might have a program to provide documents and service in the area of psychology. A program for a university library whose program structure is defined by library functions might have a program for publications, advertisements and exhibits. A university library whose program structure is oriented in terms of its organization might have a program for its engineering library. Finally, a university library might have a program structure defined along a combination of dimensions. For example, the library might have a program geared to meeting the needs of undergraduates and another program for Southeast Asian Studies. In this case, the overlap caused by the intersection of these two programs would have to be resolved to maintain the non-overlapping aspect of programs.

## A Proposed University Library Program Structure

In this section a typical program structure is developed for a university library with the initial program divisions being made largely along functional lines. These are six programs in all, five of which are user-oriented programs. The last program is composed of activities which are not easily related to user service.

Program I is directed toward providing space and furnishings for use by library patrons (student and faculty). At the subprogram level, provision of user area and furnishings are considered by library units (i.e., departmental libraries, etc.). Provision and maintenance of physical facilities for area utilized for document storage and for areas used by library staff is not considered part of Program I.

Providing access to documents within the library is the substance of Program II. The subprogram of managing document collections is further subdivided in terms of the subject categories defined in Appendix A. The subprograms to control use and location of documents and to facilitate document use are subdivided on a library unit basis. Providing indexes and catalogs are not considered part of this program. These activities appear in Program IV.

Program III deals with activities necessary to provide access to documents in other libraries. This program, except under special circumstances, would not normally be subdivided by library units.

Program IV is concerned with providing aids to assist patrons in identifying and locating documents to satisfy an information need. In addition, the service of providing personal assistance for document identification and location and that of providing information is included here. These services are defined as subprograms on a library unit basis with two exceptions. Assistance in compiling an inter-library catalog of holdings would be provided on a library-wide basis. Further, provision of indexes might be accomplished on a subject category or group of subject categories basis.

Promotion of library use comprises Program V. The subprograms of publications, advertisements and exhibits, and personal communication would be further divided by library units.

The last program, Program VI, is composed of library-wide administrative and planning activities, support activities, and the provision and maintenance of physical facilities utilized by the library staff. This program structure is presented in outline form as Appendix B.

It may be noted that this program structure cuts across organizational lines in some instances. As indicated previously, this is not important. What is important is to be able to determine benefits imparted to users in relation to resource inputs. In addition, use of the crosswalk technique allows administrators to monitor library activities in the context of both an organizational as well as program framework.

Thus far we have been concerned with the organization and arrangement of library activities into a structure which is conducive to rational decision making. Now we tackle the problem of devising a means for allocating funds within this structure in an optimal manner. We begin this task by exploring the concept of benefit-cost analysis.

## Benefit-Cost Analysis

Benefit-cost analysis is the process of relating the benefits imparted by a particular activity to the costs of performing that activity. In this research the implicit assumption is made that the measure of expected future weighted exposures, discounted over time, is the measure of expected future benefits denoted by B. A measure of the current library resources and expected future program costs, discounted over time and adjusted for inflationary effects, comprises the cost associated with each program, for simplicity of notation symbolized by C.

If we let

$\Delta B_i$ = incremental number of future weighted exposures as a result of an incremental change in the expenditure level of program i

$\Delta C_i$ = total incremental expected costs of operations and services, discounted over time and adjusted for inflationary effects, as a result of an incremental change in the expenditure level of program i

then $\frac{\Delta B_i}{\Delta C_i}$ represents the marginal change in benefits per unit cost resulting from a change in the level of program i. If funds are allocated to n programs such that

$$\frac{\Delta B_1}{\Delta C_1} = \frac{\Delta B_2}{\Delta C_2} = \cdots = \frac{\Delta B_n}{\Delta C_n}, \tag{3.1}$$

and if these n programs represent the universe of relevant library programs, then an optimal allocation of funds has been achieved. This analysis should be extended to include the subprograms as well.

To see why benefits are maximized when funds are allocated among the n programs in this fashion, we consider the simple case in which n = 2. If we let

C = total funds to be allocated among the two programs,

$B_i$ = units of expected benefits yielded by the $i^{th}$ program,

$x_i$ = units of input purchased for program i,

and $p_i$ = cost per unit of input for program i,

then we wish to

$$\text{maximize } Z = B_1 + B_2 \qquad (3.2)$$

$$\text{subject to: } p_1x_1 + p_2x_2 \le C \qquad (3.3)$$

We assume that the cost per unit of input for each program is constant, which seems to be a reasonable assumption within the relative range of library operations. We further assume that $B_i$ is a monotonic increasing function at a decreasing rate with $x_i$ as seen in Figure 3.1 within the relevant range of library operations. This seems to be a reasonable assumption as we would expect initial expenditures would be for programs yielding the highest ratio of benefits to expenditures. Then as more funds are available, those programs which yield the next highest ratio of benefits to expenditures are funded, and so on.

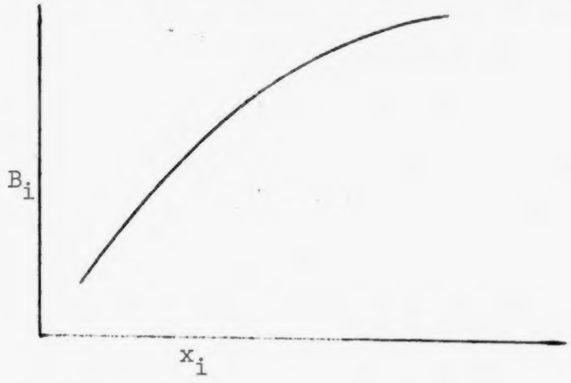

Figure 3.1--Relationship Between the Units of Benefits, $B_i$, Yielded as a Function of the Units of Input, $x_i$, For Program i.

Solution of equations 3.2 and 3.3, using the technique of Lagrangian Multipliers, proceeds as follows:

$$\text{maximize } L = B_1 + B_2 - \varphi(C - p_1x_1 - p_2x_2) \tag{3.4}$$

$$\frac{\partial L}{\partial x_1} = \frac{\partial B_1}{\partial x_1} + \varphi p_1 = 0 \tag{3.5}$$

$$\frac{\partial L}{\partial x_2} = \frac{\partial B_2}{\partial x_2} + \varphi p_2 = 0 \tag{3.6}$$

Simultaneous solution of equations 3.5 and 3.6 yields

$$\frac{\partial B_1}{p_1 \partial x_1} = \frac{\partial B_2}{p_2 \partial x_2} \tag{3.7}$$

This indicates that an extreme has been reached at the point wherein the incremental change in $B_1$ divided by $p_1$ times the incremental change in $x_1$ equals the incremental change in $B_2$ divided by $p_2$ times the incremental change in $x_2$. Further,

$$\frac{\partial^2 L}{\partial x_1^2} = \frac{\partial^2 B_1}{\partial x_1^2} = \text{negative value} \tag{3.8}$$

and $$\frac{\partial^2 L}{\partial x_2^2} = \frac{\partial^2 B_2}{\partial x_2^2} = \text{negative value}, \qquad (3.9)$$

as the relationship between $B_i$ and $x_i$ is increasing at a decreasing rate, indicating the solution is for a maximum point.

A similar analysis could be performed for the case in which $B_i$ and $x_i$ are expressed in discrete units. In this case the solution would appear as

$$\frac{\Delta B_1}{p_1 \Delta x_1} = \frac{\Delta B_2}{p_2 \Delta x_2} \qquad (3.10)$$

This analysis could be extended to the case in which funds are to be allocated among n programs. In this case the resulting solution would appear as in equation 3.1, where $\Delta C_i = p_i \Delta x_i$. This scheme of distributing funds to programs such that the ratios of marginal benefits-to-expenditures are equal for all programs appears to make sense. For if the marginal benefit-to-cost ratio for one program were higher than the marginal benefit-to-cost ratio of other programs, would it not make sense to transfer a sum of funds from these other programs to the program with the higher ratio? In this manner a greater number of benefits would accrue as the result of investing this sum of funds in the program with the higher benefit-cost ratio. Further, would it notseem desirable to continue this process of transferring funds to the program or programs with the higher marginal benefit-to-expenditure ratios until no further increase in the overall level of benefits could be achieved? The point at which this transfer of funds from one program to another yields no further increase in benefits, is the point at which the values of the marginal benefit-to-expenditure ratios are the same for all programs.

A special case arises when programs are encountered which can be funded only at a particular level. An example of this might involve membership in an inter-library cooperative system, wherein unlimited service is offered for a fixed fee. In this situation the relevant value of the ratio of marginal benefits per expenditure is identical to the value of the ratio of total benefits per expenditure associated with the decision to participate in the program. Obviously the value of the marginal benefit-to-expenditures ratio is zero for expenditures less than the requirements of the program and is zero for expenditures in excess of the requirements of the program. In this case a decision should be made to participate in the program if the relevant benefit-to-expenditure ratio equals or exceeds the marginal benefit-to-expenditure ratio of other programs.

Library administrators might desire to exclude certain programs (subprograms) from this analysis for a variety of reasons. To accomplish this, funds allocated to the excluded programs would be viewed as fixed requirements. These programs would be funded first, with the remaining funds allocated among the remaining programs. An example of this situation entails the allocation of a fixed sum of money for administrative services. Another example arises when a library administrator prefers to allocate a fixed sum of money for a specialized program e.g., procurement of rare books. It is not that benefit-cost analysis does not apply, but rather that the construction of benefit-cost indicators for use in the analysis is difficult for these cases. For example, undoubtedly, up to a point, the more spent on personnel administration, the higher is employee morale, which indirectly affects the number of exposures in a positive manner. Or the more expended for planning, the more effective is library management which results in greater library effectiveness. Also, the more spent on the procurement of rare books, the "better" the rare book collection resulting in increased benefits to the university community. Although it is extremely difficult to assess the benefits resulting from these types of programs, where possible, indicators should be constructed to inform the library administrator the price he is paying for the outputs obtained from those programs (subprograms) excluded from the benefit-cost analysis. For example, in the case of rare book procurement, an unweighted measure of the expected future exposures per expected future cost might be of value in deciding upon the amount of funds to be spent on the procurement of rare books.

## Funding Allocation

Ultimately decisions are made which determine the manner in which funds are to be allocated among the various programs. The determination of how funds are to be distributed results from decisions on the part of library administrators regarding the types of service and level of service to be offered. The type and the level of service offered are viewed as the decision or controllable variables. In this context, the term "service" is used to describe outputs of programs which users of the library contact directly. The types and level of service offered affects the degree of user activity which in turn affects the degree of document exposure. This can be shown symbolically as follows. If we let

$\Delta S_i$ = incremental change in the level of service of program i (program or subprogram)

$\Delta C_i$ = incremental change in costs directly associated with a change in level of service of program i

$\Delta A_i$ = incremental change in user activity associated with a change in the level of service of program i

$\Delta E_i$ = incremental change in the number of exposures associated with a change in the level of service of program i

then $\Delta C_i \longrightarrow \Delta S_i \longrightarrow \Delta A_i \longrightarrow \Delta E_i$

where $\longrightarrow$ is interpreted as "affects," "influences," or "causes" a change.

$\Delta C_i$ is defined as an incremental change in funding which, when applied directly to service activity of program i, results in a change in the number of exposures $\Delta E_i$. In some instances this change in the number of exposures, $\Delta E_i$, as a direct result of a change in the level of service activity of program i, $\Delta S_i$, will necessitate a change in the level of service activity associated with program k, $\Delta S_k$. This phenomenon occurs as a result of the interrelationships encountered between library activities making it virtually impossible to define programs which are completely independent of one another. This change in service, $\Delta S_k$, will in turn necessitate a change in the requirements for funds, $\Delta C_k$, in support of this service. Hence, we have

$$\Delta E_i \longrightarrow \Delta S_k \longrightarrow \Delta C_k'$$

where $\Delta C_k'$ = change in funding requirement for service activity of program k as a result of a change in the level of service activity of program i. An example of this phenomenon occurs when additional funds are expended to increase the number of aids available for identifying existing documents. As a result of this increase, there would be an increase in the number of documents consulted (exposures). However, the greater the number of documents consulted, the greater the level of service required in the area of document control (circulation, in-library use, periodical control, etc.), which in turn requires an increase of funds to accommodate this increase in document control service. In general we would expect that an increase in any service activity, designed to increase the number of exposures, will cause an increase in document control service requiring an increase in funds.

Therefore, in constructing benefit-cost ratios for determining the marginal benefits per incremental change in cost inputs, this cost measure should include a figure for both the change in costs directly associated with the service activity being altered as well as the change in costs associated with other service activities affected by this action. Symbolically, this may be represented as follows:

$$\Delta C_i + \sum_k \Delta C'_k$$

where $\Delta C'_k$ is summed for all programs affected by a change in the level of the service of program i.

In the construction of these models care must be taken to make them adaptive to changes in the variables affecting document exposure. For example, the composition and size of the university community is of critical concern when formulating these variables into the various models.

## Summary

In this chapter the essential elements of PPB as a management system were presented. Four dimensions for classifying library activity into virtually mutually exclusive and exhaustive subsets were discussed. The dimensions discussed were population, documents, library functions, and organizational units. A typical program structure was constructed designed primarily along functional lines. This does not imply that the proposed program structure is necessarily the only or best program structure for a university library, rather that it is one of many program structures which seem conducive to the construction and use of benefit-cost ratios in the PPBS management process.

The concept of benefit-cost analysis was discussed with particular emphasis placed upon its usefulness in the decision-making process for determining the optimal allocation of funds to the various programs (subprograms). It was observed that the library attains maximum effectiveness when

$$\frac{\Delta E_1}{\Delta C_1} = \frac{\Delta E_2}{\Delta C_2} = \cdots = \frac{\Delta E_n}{\Delta C_n}$$

for each of the programs (subprograms) offered by the library. In the construction of these ratios care must be exercised to be certain that the incremental changes in exposures and costs are attributed to the programs (subprograms) whose level of service are altered. In addition, these ratios should be constructed to account for any future change in exposures, discounted over time, and any future changes in costs, adjusted for inflation and the discount rate of money, as developed in Chapter II.

Ideally it is desirable to identify the functional relationships between the variables of expenditures, level of service, user activity, and exposures for each program. Given the nature of these relationships, it is then possible to predict changes in the process,

$$\Delta C_i \longrightarrow \Delta S_i \longrightarrow \Delta A_i \longrightarrow \Delta E_i,$$

as a result of employing alternative strategies. Values of the ratio of marginal benefits to marginal expenditures can then be estimated corresponding to alternative levels of service of the program (subprogram). In addition, it may be necessary to explore the relationships descriptive of the secondary effects whereby

$$\Delta E_i \longrightarrow \Delta S_k \longrightarrow \Delta C'_k ,$$

when increased activity in one program necessitates an increase in service activity in one or more related programs. This will insure that proper charges will be allocated appropriate service activities affecting the change in document exposure.

To the extent possible, analytical models can be developed descriptive of these relationships. Then, utilizing these models, changes in the number of exposures per change in expenditures can be estimated as a function of alternative levels of service. This output enables the library administrator to determine the effects of alternative levels of service on document exposure for the programs within library budget restrictions. Using the resulting marginal benefits-to-marginal exposure ratio, the library administrator can determine the allocation of resources which leads to greater document exposure. Examples of these analytical models are included in Chapter IV.

# CHAPTER IV

# ANALYTICAL MODELS FOR DECISION MAKING

## Introduction

In this chapter analytical models are constructed to assist in decision making for selected subprograms. No attempt is made to present a comprehensive set of analytical models corresponding to each subprogram. To develop such a set would require years of intensive research especially in the construction of models for some of the less structured subprogram areas. In general the goal of each model is to yield a benefit-cost ratio, in the form of $\Delta B/\Delta C$, for predicting effects of employing various types of strategies within a subprogram. In constructing a model for a subprogram the relationship is determined between the controllable or decision variables and the benefits yielded by the subprogram. Utilizing the model, it is then possible to predict changes in the value of $\Delta B/\Delta C$ as a function of changes in the value of the decision or controllable variables for the subprogram.

Most previous attempts to construct analytical models in these areas have conceptualized the library as a closed system. Few models have been developed which are sensitive to changes in the composition and size of the university community which would seem to play a major role in influencing demand for library service. To the extent possible these factors are considered in the construction of analytical models in this research.

In this chapter analytical models are constructed to provide guidance for administrators in making decisions relating to future demands for service, the selection of multiple copies, the selection of new titles, and reserve service. Specifically, a Circulation Demand Model is developed for estimating the demand for circulation use and in-library use of titles as a function of actual circulation and variables descriptive of the teaching and research efforts of the university. A Multiple Copy Selection Model and New Title Selection Model are developed to provide guidance to the administrator in determining effective strategies for the purchase of multiple copies and new titles respectively. Finally, a model useful for assisting in the decision-making process for determining the number of copies of a title to place on reserve is constructed. In each case the decision variables are identified and guidelines presented for the construction of the relevant benefit-cost ratios for use in the decision-making and planning process.

### Circulation Demand Model

In the program designed to provide access to documents within the library, a rather significant subprogram involves the selection of new titles and multiple copies for titles within each subject area. In this instance we are considering only titles which are permitted to circulate. This excludes books which are on reserve, books restricted to in-library use, periodicals and indexes. Ultimately it is desirable to construct a model which will assist the library administrator in determining the amount of funds to allocate for the purchase of new titles and multiple copies within each of the various subject areas. Thus, within each subject area, the decision variables include the number of new titles and the number of copies of each title to purchase each year. We assume that within a given subject area the optimal book purchase strategy is reached when the marginal benefit-cost ratio of adding an additional copy equals the marginal benefit-cost ratio of adding an additional title. This is a logical extension of the objective developed in Chapter II, to maximize the expected number of document exposures. The optimal allocation of funds among subject areas is determined when these benefit-cost ratios are equal for all subject areas. Hence,the optimal amount of total funds for the purchase of new titles and multiple copies for circulation use for all subject areas is specified, when the marginal benefit-cost ratios for purchasing an additional title or an additional copy of a title equal the marginal benefit-cost ratios for the expenditure of additional funds in other programs. These benefit-cost ratios for adding a title and adding an additional copy of a title provide a basis for determining an effective allocation of funds within the library.

In this section a model is developed which can be used to predict demand for titles which are permitted to circulate. As this model predicts demands as a function of the size and composition of the university community, it will be a valuable aid in providing forecasts of future demand for services for use in the planning process. In addition, the output of this model can be used in the computation of the marginal benefit-cost ratios to evaluate alternative strategies for the purchase of new titles and multiple copies for each subject area.

### Background

Undoubtedly the most significant model-building effort in this area is Philip Morse's Circulation Inference Model.[1] In this model the number of times a book is demanded is assumed to be described by Poisson distribution with $\lambda$ defined as the mean number of times a year users desire to borrow a particular book. Since the value of $\lambda$ cannot be determined directly from past circulation data, a model is constructed to estimate $\lambda$ based upon the actual number of times a book circulates each year. Here demand rate is defined as the number of times a document is requested during a specified period of time. Circulation rate is defined as the number of times a document is actually withdrawn during a specified period of time. $\mu$ is defined as the mean number of loan periods per year or the rate at which a title is returned after being circulated. $^1/_\mu$ is defined as the mean loan period representing the time duration between the act of removing the book to be borrowed from the shelf to the act of replacing the book on the shelf ready for use by the next person. The actual loan period for any given book is assumed to be distributed in the form of an exponential distribution.

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[1] Philip Morse, Library Effectiveness: A Systems Approach, p.123-131.

In the Morse Model the demand rate for circulation use, $\lambda$, for a particular document is assumed to be a random variable described by an exponential distribution with unknown mean demand rate, $\overline{D}$. Hence, $P(\lambda)$ equals the probability a document will be demanded at the rate of $\lambda$ times per year where

$$P(\lambda) = \frac{1}{\overline{D}} e^{-\lambda/\overline{D}} \tag{4.1}$$

$P(m|\lambda)$ is defined as the probability that a particular document circulates m times in a year given that it is demanded at a rate $\lambda$.

The distribution $P(m|\lambda)$ is assumed to be Poisson. The mean of this distribution is computed utilizing a queuing model which assumes that demands for documents temporarily not in the library are permanently lost. In the solution of this queuing model it can be shown that $\mu/(\mu + \lambda)$ is equal to the probability that a particular book is available in the library. $(\lambda)(\mu)/(\lambda + \mu)$, the demand rate times the probability the book is available, equals the circulation rate of a particular book and is the mean of the conditional Poisson distribution, $P(m|\lambda)$. The joint distribution $P(m, \lambda) = P(m|\lambda)\ P(\lambda)$, when integrated over $\lambda$, yields the unconditional distribution $P(m)$ in terms of $\overline{D}$, the mean demand rate of all documents. Setting the expected value of m equal to the actual average circulation rate, $\overline{R}$, establishes an equation which can be solved for $\overline{D}$, the mean demand rate for documents. The value of $\overline{R}$ is determined from circulation history of sampled books. Morse obtains the following solution to this equation via some approximation methods.

$$\overline{D} \approx 2.5\,\mu \left(\frac{\overline{R}}{\mu}\right)^{2.06} + \overline{R}$$

Hence, given values for the mean circulation rate, $\overline{R}$, and of the mean loan period, $^{1}/\mu$, estimates can be made of the mean demand rate, $\overline{D}$, and of the mean unsatisfied demand rate, $\overline{D}$-$\overline{R}$, for a class of books.

At this point we note that $\overline{R}$ cannot be simply equated with $(\lambda)(\mu)/(\mu+\lambda)$ to solve for $\lambda$. Since the value of $\lambda$ is a random variable for any given document, it is not permissible to solve a deterministic expression to obtain the mean of this random variable. Hence a distribution must be assumed for $\lambda$ and the solution must proceed in the manner previously outlined.

The limitations of this model for predicting the number of satisfied and unsatisfied demands in a given subject area during any given period are as follows.

1. Future demand for circulation service is assumed to be solely a function of past circulation use. The model is not sensitive to changes in the magnitude and composition of the teaching and research efforts of the university community which undoubtedly affect demand.

2. This model applies only to unduplicated books. In a library system in which a substantial amount of duplication is present, this model would yield questionable results.

3. The demand rate, $\lambda$, for a given book is assumed constant over time. This model would yield questionable results in situations in which the demand rate varies as a function of time.

Nevertheless, Morse's efforts represent a major contribution in the modelling of the book circulation process.

Fussler and Simon investigated circulation use of various documents in specific subject categories at different universities as a function of publication date, accession date, language, use in past five years, years since last use, and various combinations of these. The purpose of this investigation was to derive a function which would "predict, with reasonable accuracy, the frequency with which groups of books with defined characteristics are likely to be used in a research library".[2] The implication is that groups of books with a low probability of future use could be stored more economically in less accessible areas without significantly impairing library service. For libraries possessing no record of past use, using rules based upon language and publication date, storage of 25 per cent of the collection would result in an inconvenience for 3 per cent of the total circulations. For libraries possessing circulation records for five years, using a rule based upon use in the past five years, storage of 25 per cent of the books in the collection would affect 2 per cent of the total circulations. Finally, if records for circulation use for 20 or more years are available, only 1 per cent of the total circulations would be affected by storing 25 per cent of the collection based upon rules utilizing time since the last circulation. Further work in this area has been performed by Trueswell.[3]

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[2] H. Fussler, J. Simon, Patterns in the Use of Books in Large Research Libraries, p.14.

A rather complete analysis of book circulation use studies and models was performed by Jain. The purpose of this study was "to examine some previously developed mathematical models and also to develop some new mathematical models and statistical techniques for studying the dependence of circulation rate on a book's age and other characteristics."[4] Four classes of book use models which predict expected future circulations are discussed. They are the exponential class, the logarithmic class, the square root class, and the Markov model class. Jain proposes an additional model, termed a "$P_n$ model", which employs a truncated Poisson distribution, having a separate higher probability for the event of zero annual demand. This analysis is performed for the purpose of determining which books might be assigned to less accessible storage areas. Further details of the "$P_n$ model" are presented in Jain, Leimkuhler, and Anderson.[5]

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[3] Richard Trueswell, "A Quantitative Measure of User Circulation Requirements and Its Possible Effect on Stack Thinning and Multiple Copy Determination," American Documentation, 16:20-25 (Jan. 1965).

[4] A. K. Jain, A Statistical Study of Book Use, p.XII.

[5] A. K. Jain, F. F. Leimkuhler, and V. L. Anderson,"A Statistical Model of Book Use and Its Application to the Book Storage Problem", Journal of the American Statistical Association 64:1211-24 (Dec. 1969).

A study conducted by McGrath[6] relates circulation use per subject category to various variables exogenous to the library. Statistically significant simple correlations were observed between the number of books circulated in each subject category and:

1. the number of books published in the U. S. per subject category;
2. the number of books held by the library in each subject category;
3. the number of faculty members in departments related to each subject category;
4. the number of course credit hours offered related to each subject category.

The findings of this study suggest that certain variables exogenous to the library related to the research and teaching functions of the university, as well as variables related to the supply of books, can be utilized to predict demand for library service.

### The Model

This model is developed in two stages. Stage I predicts the current demand rate for documents as a function of the current circulation rate of documents by subject areas. Obviously demand rate will exceed circulation rate because a certain portion of demand is for documents which are currently out on loan. Stage II is addressed to ascertaining the relationship between demand rate and the teaching and research activity of the university community.

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[6] W. McGrath, R. Huntsinger, and G. Barber, "An Allocation Formula Derived from Factor Analysis of Academic Departments," College and Research Libraries, 30:533-539 (Sept. 1969).

This model will permit a library administrator to predict the demand rate for books within each subject area as a function of the teaching and research activity for use in the planning process. In addition, the output of this model is used as input to models for evaluating different strategies governing the purchase of new titles and multiple copies. This model will differ from the Morse model previously described in the following ways.

1. Weekly circulation data will be utilized which are readily obtainable as a by-product of the circulation service administrative process. Morse determines the average number of circulations per year per document by sampling documents and obtaining past circulation histories from the back fly leaf.
2. Multiple copies will be considered in this analysis.
3. Expected demand for circulation use will be related to the teaching and research activities of the university making it possible to predict future demand for circulation as the structure and composition of the university change.
4. Demand rate as a variable function of time will be considered.

### Stage I

The general approach for constructing a model for predicting current demand rate based upon current circulation rate in a particular subject area is now described. Initially, for simplicity, let us assume that the demand rate for titles which circulate is constant over time. Of course, this assumption is not valid since the demand rate for books is noticeably less at the beginning of a semester than at a point, say midway in the semester. Nonetheless, this assumption will simplify matters for the present.

Let us define the following functions for a particular document

$\lambda$ = demand rate for circulation use of a particular title

$P(\lambda)$ = probability that the title is demanded at rate $\lambda$

$c$ = the number of multiple copies of the title

$P(c|\lambda)$ = the probability that there are c multiple copies of the title, given there is a demand rate $\lambda$ for the title

$P(S|c,\lambda)$ = the probability that at least one copy of the title is on the shelf available for circulation given there are c copies of the title and the demand rate for the title is $\lambda$

Further, we let

$N_i$ = the total number of titles which are allowed to circulate in subject area i

$G_i$ = the number of documents which circulate per time period in subject area i

Then the probability that at least one copy of a particular title, with demand rate $\lambda$, is available for circulation in a given time period is

$$P(S|\lambda) = \sum_{c} P(S|c,\lambda)P(c|\lambda) \tag{4.3}$$

The circulation rate experienced by a particular title, with demand rate $\lambda$, is equal to the probability that at least one copy of the title is on the shelf times the demand rate, i.e., $P(S|\lambda)(\lambda)$. Therefore, for all documents within a given subject area which are permitted to circulate, the number of circulations per time period can be described by:

$$G_i = N_i \int_{o}^{\infty} \lambda P(S|\lambda)P(\lambda)d\lambda = N_i \int_{o}^{\infty} \sum_{c} \lambda P(S|c,\lambda)P(c|\lambda)P(\lambda)d\lambda \tag{4.4}$$

Hence, given periodic circulation data, $G_i$, and an estimate of the total number of titles in a given subject area, $N_i$, this expression can be solved to yield an estimate of the mean demand rate, $\overline{D}_i$, for titles within that subject area. To carry out the solution, probability distributions must be specified for $P(S|\lambda,c)$, $P(c|\lambda)$ and $P(\lambda)$. This problem is considered next.

Studies by Morse,[7] Fussler and Simon,[8] Jain,[9] and some of our preliminary empirical findings indicate that a great many titles are infrequently demanded whereas a small percentage of titles experience a high demand rate. In addition, a relatively high percentage of titles experience no demand for circulation whatsoever. As indicated earlier in this chapter, Jain developed the $P_n$ model using a modified Poisson probability function with a disproportionately higher probability assigned to zero uses during a given year, to describe this phenomenon. A similar probability function is utilized here for the case of a continuous random variable. In this case an exponential distribution is employed, augmented to include a discrete probability for the case of demand rates of zero. An exponential distribution is employed as it attributes a low probability of occurrences to high values and a high probability of occurrences to low values which seems descriptive of the demand rate experienced by titles. In addition, its continuous nature makes the solution of the model more tractable. Therefore, the probability function which is used to describe the distribution of demand rate, λ, for any particular document is as follows:

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[7] Morse, Library Effectiveness, p. 126.

[8] Fussler and Simon, Patterns in the Use of Books, p. 145.

[9] Jain, A Statistical Study of Book Use, p. 24.

$$P(\lambda) = \begin{cases} p_o & \text{for } \lambda = 0 \\ \dfrac{(1-p_o)e^{-\lambda/\overline{D}_i}}{\overline{D}_i} & \text{for } 0 < \lambda < \infty \end{cases} \qquad (4.5)$$

where $p_o$ = the probability that a title experiences zero circulations and $\overline{D}_i$ = estimated mean demand rate for titles which circulate in subject area i.

The magnitude of $p_o$ can be estimated by sampling books within a given subject area and determining the proportion of these books that have experienced no circulation.

We assume that there exists a relationship between the number of multiple copies of a particular document and the demand rate for the document. This relationship is expressed in the probability function $P(c|\lambda)$. We assume this relationship exists as a result of judicious purchases of multiple copies based upon the experience and foresight of those personnel selecting books. In addition, most libraries have policies regarding the purchase of additional copies if patrons register a certain number of reserve requests when all copies of the document are out on loan. This results in further strengthening the relationship between demand rate for titles and the number of multiple copies. In general we expect that if the demand rate is low, there is a small probability that there is more than one copy of the title. On the other hand, if the demand rate is high, we expect that the probability is high that there is more than one copy of the title. Thus, the form of the conditional distribution, $P(c|\lambda)$, might appear somewhat as depicted in Figure 4.1 for two different values of $\lambda$.

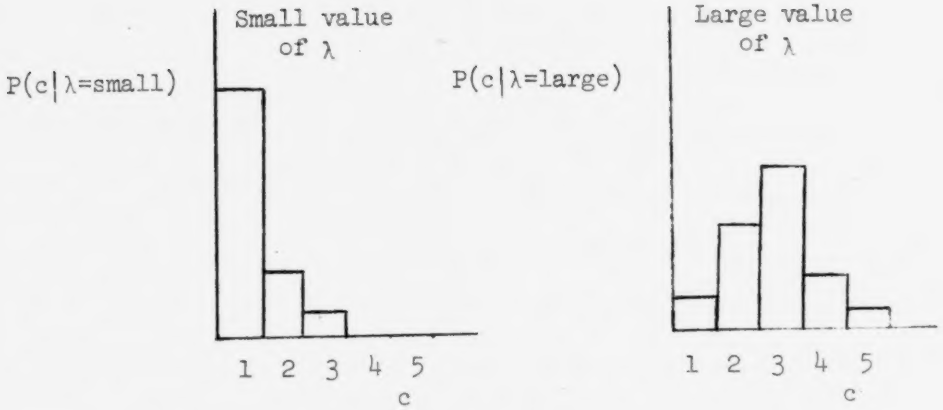

Fig. 4.1--Conditional Probability Distribution for the Number of Copies of a Title Given a Particular Demand Rate.

To determine the form of this distribution, descriptive of this relationship, data on the number of circulations and the number of multiple copies can be determined for a sampling of books. A probability function can be fit to the data descriptive of the relationship between multiple copies of a title and the circulation rate of the title. This analysis is discussed in Chapter V. We assume that the relationship between the number of multiple copies and the demand rate can be approximated by the relationship between the number of multiple copies and the circulation rate for titles in the collection. This assumption is necessary as it is virtually impossible to collect demand rate data for individual documents unless the library utilizes a "closed stack" arrangement.

Finally, it remains to specify the function describing the probability that at least one copy of a particular title is available for circulation use given that there are c copies and that the title experiences demand rate $\lambda$. To accomplish this a queuing model is utilized assuming a demand rate (arrival rate) $\lambda$, and use period (service time) $^1/\mu$. Use period ($^1/\mu$) is defined as the time period which begins when a document is removed from the shelf by a patron and ends when the document is returned to the shelf. To simplify the model, let us assume that, if a patron is unable to obtain a document because it is out on loan, the patron does not attempt to obtain that particular document again. If we let

$P_n$ = probability that n copies of a document are out on loan for $0 \le n \le c$.

Then,

$$\begin{aligned} P'_o &= -\lambda P_o + \mu P_1 && n = 0 \\ P'_n &= -\lambda P_n - n\mu P_n + \lambda P_{n-1} + (n+1)\mu P_{n+1} && 1 \le n < c \qquad (4.6) \\ P'_c &= -c\mu P_c + \lambda P_{c-1} && n = c \end{aligned}$$

The steady state solution yields:

$$P_n = \left(\frac{\lambda}{\mu}\right)^n \frac{P_0}{n!} \qquad 0 \le n \le c \qquad (4.7)$$

We know that

$$\sum_{n=0}^{\infty} P_n = 1 \qquad (4.8)$$

therefore,

$$P_0 = \frac{1}{\sum_{n=0}^{c} \left(\frac{\lambda}{\mu}\right)^n \Big/ n!} \qquad (4.9)$$

The probability that at least one copy of a document is available for circulation given $\lambda$ and c, is as follows:

For c = 1: $P(S|\lambda,c) = P(n < 1|\lambda,c) = 1 - P_1 = 1 - \dfrac{\lambda/\mu}{1 + \lambda/\mu}$

For c = 2: $P(S|\lambda,c) = P(n < 2|\lambda,c) = 1 - P_2 = 1 - \dfrac{\lambda^2/2\mu^2}{1 + \lambda/\mu + \lambda^2/2\mu^2}$

For c = 3: $P(S|\lambda,c) = P(n < 3|\lambda,c) = 1 - P_3 = 1 - \dfrac{\lambda^3/6\mu^3}{1 + \frac{\lambda}{\mu} + \frac{\lambda^2}{2\mu^2} + \frac{\lambda^3}{6\mu^3}}$

(4.10)

etc.

The value of $\mu$ can be estimated for the use of documents in each subject area by utilizing a technique presented by Morse.[10] If we let

$N'_i$ = total number of documents in a given subject area,

$L_i$ = number of documents out on loan at any point in time in a given subject area

then,

${}^{L_i}/_{N_i}$ = proportion of documents out on loan in the subject area at any given time.

Further, if

${}^{1}/_{\mu_i}$ = use period of a document in subject area i,

$\bar{R}_i$ = mean circulation rate of documents in the subject area i (this can be estimated from documents which were sampled to verify the modified exponential distribution, $P(\lambda)$),

then,

${}^{\bar{R}_i}/_{\mu_i}$ = proportion of time the average document is out on loan.

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[10] Morse, Library Effectiveness, p.120

Therefore,

$$\frac{\overline{R}_i}{\mu_i} = \frac{L_i}{N_i'}$$

and (4.11)

$$\mu_i = \frac{\overline{R}_i N_i'}{L_i}$$

But $\overline{R}_i N_i = G_i$, where $G_i$ = number of circulations during a specific period of time in subject area i. Therefore

$$\mu_i = \frac{G_i}{L_i} \qquad (4.12)$$

If we can assume that the mean use period is similar for each subject area then $\mu$ can be estimated by $^G/L$, total number of circulations divided by the total number of documents out on loan.

This concludes the formulation necessary to obtain the mean demand rate, $\overline{D}_i$, for documents in subject area i as a function of the recorded circulations, $G_i$, in the subject area. From this analysis it is also possible to estimate the number of unsatisfied demands (demands to take a book out on loan when all copies of the book are currently out on loan).

Therefore, given the following model

$$G_i = N_i \int_0^\infty \lambda \sum_c (S|c,\lambda)P(c|\lambda)d\lambda$$

$$G_i = \frac{(1-p_o)(N_i)}{\overline{D}_i} \int_0^\infty \lambda \Big[ (1 - \frac{\lambda}{\lambda+\mu}) P(c=1|\lambda)$$

$$+ (1 - \frac{\lambda^2}{2\mu^2+2\mu\lambda+\lambda^2}) P(c=2|\lambda)$$

$$+ (1- \frac{\lambda^3}{6\mu^3+6\lambda\mu^2+3\lambda^2+\lambda^3}) P(c=3|\lambda) + \ldots \Big] e^{-\lambda/\overline{D}_i} d\lambda \qquad (4.13)$$

and having data for $G_i$, $N_i$, $\mu$, $P(c|\lambda)$, and $p_o$, we can solve for $\overline{D}_i$, the mean demand rate for circulation use of documents within a particular subject area.

## Stage II

Now that a means has been devised to predict demand for circulation use as a function of actual recorded circulation use, the model is developed further to relate demand for documents in a particular subject area to the teaching and research activity of the university. To accomplish this, a multiple regression analysis can be performed relating the dependent variable, $\overline{D}_i$, to a set of independent variables, $X_{ij}$. Hence,

$$\bar{D}_i = f(X_{i1}, X_{i2}, \ldots, X_{in})$$

where $\bar{D}_i$ = mean demand rate for the circulation use of titles in subject area i

$X_{ij}$ = $j^{th}$ variable describing an aspect of the teaching and research activity of the university which is related to subject area i.

The specific $X_{ij}$ variables are defined in terms of the subject areas presented in Appendix A.

To perform a regression analysis it is desirable to have a number of data points describing the same interaction over a period of time (e.g., average circulation data, a count of the number of documents in each subset area, population subset data, etc. for each semester for a number of years). Since these data are generally unavailable, the multiple regression analysis is conducted using cross-sectional data with a subject area used as the unit of association. Subject areas can be grouped into broad categories according to similarity of book material and according to predicted similarity of behavior exhibited by patrons using these documents. Approximately 30-40 subject areas can be grouped according to each of the following broad categories:

1. Engineering
2. Physical Science
3. Business-Economics-Urban Studies
4. Social Science
5. Humanities

A separate regression analysis can be performed for each broad category to determine the parameters useful in predicting future demand for documents based upon the variables descriptive of the teaching and research activity of the university.

This concludes the two stage presentation of a model which can be used to

1. estimate the demand rate for the circulation use of titles as a function of circulation use data and,
2. predict the demand rate for circulation use of titles within a given subject area as a function of variables describing the teaching and research activity of the university, for titles which experience a constant demand rate throughout the year.

### Time Dependent Demand Rate

The preceding analysis pertains to the case in which demand rates for individual books are constant over time. Weekly circulation data for libraries on the University of Pennsylvania campus indicate a definite seasonal fluctuation during the period of a semester. The number of circulations per week increases reaching a maximum approximately two-thirds of the way through the semester. During the latter one-third of the semester, the number of circulations per week decreases. Examples of this pattern are shown in Figures 4.2 and 4.3 for Van Pelt and Lippincott Libraries during the Spring Semester of 1970.

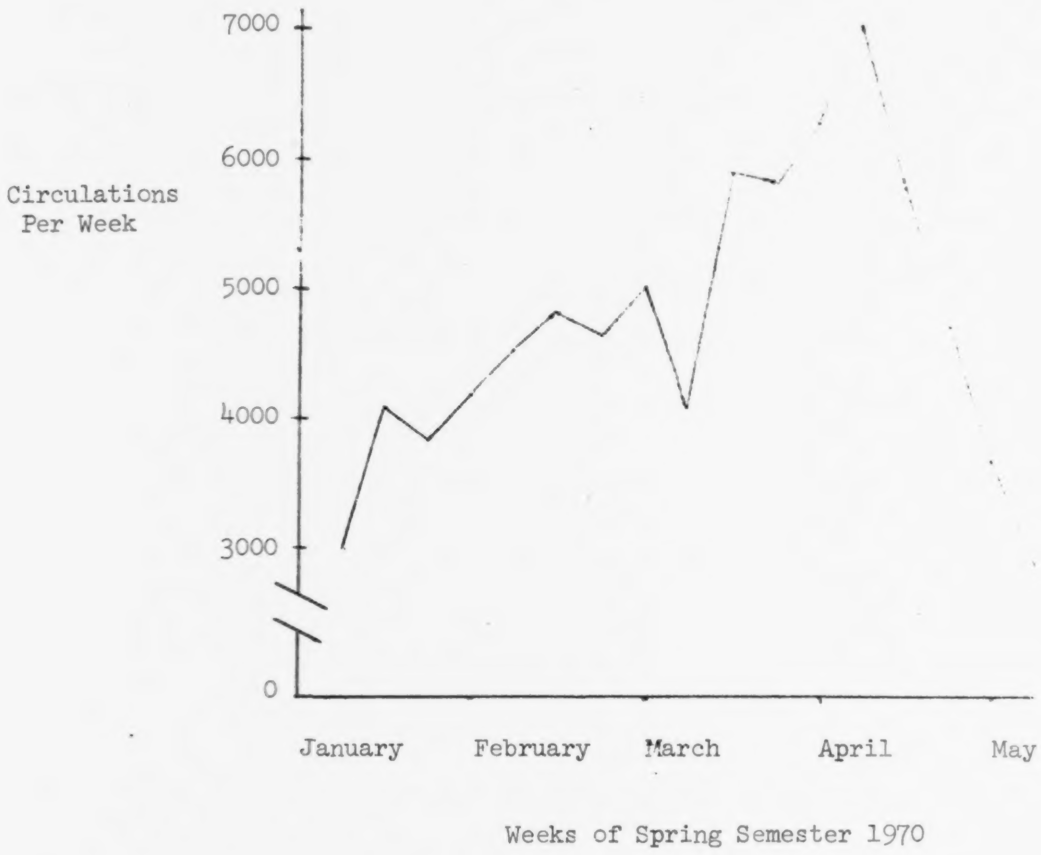

Fig. 4.2--Circulations per Week for Van Pelt Library During Spring Semester of 1970.

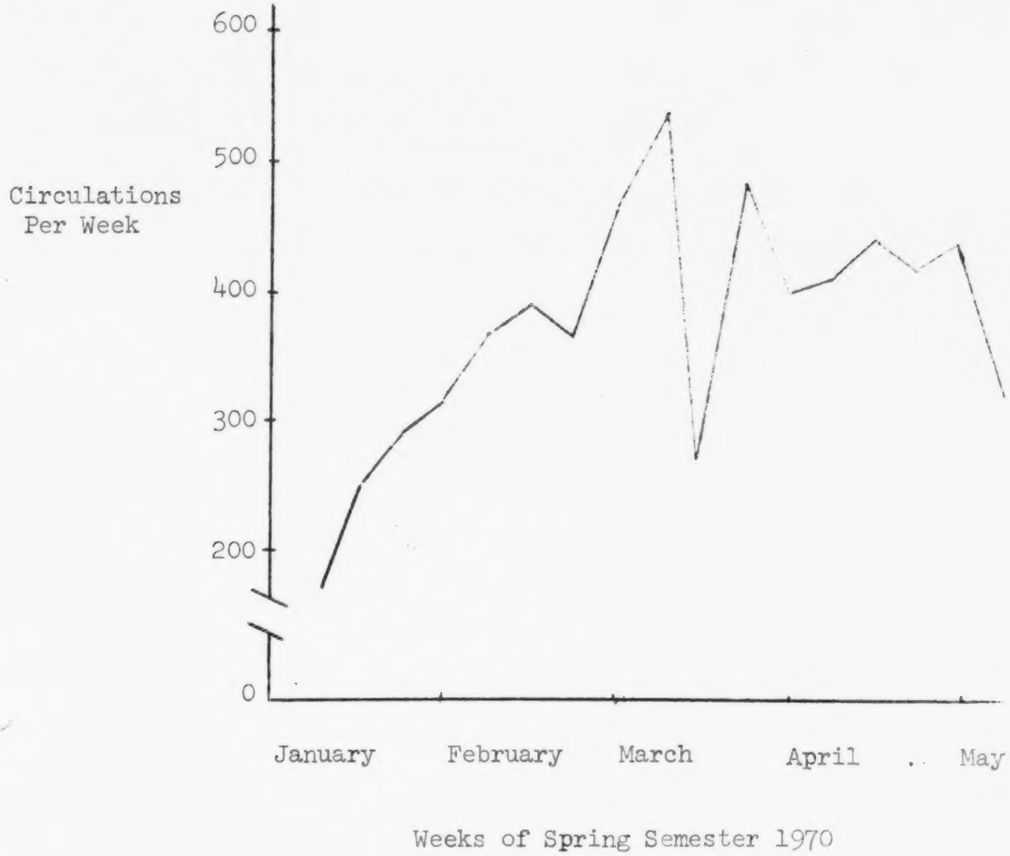

Fig. 4.3--Circulations per Week for Lippincott Library During Spring Semester of 1970.

This pattern would seem to coincide with one's intuitive feelings concerning user demand for documents during a semester, especially as related to the teaching program of the university. Initially we would expect a relatively low demand rate which increases steadily as students prepare papers and study for examinations as the semester progresses. Near the end of the semester we would expect that most students have obtained the necessary books for accomplishing their tasks, resulting in a declining demand rate during this period. We would expect that this pattern would be duplicated at most universities.

A week is selected in this analysis as a basic time period for data collection. The use of weekly demand rate data accounts for this seasonal variation experienced by titles during the semester yet alleviates problems arising from daily variations in demand.

### Exact Analysis

The use of a time dependent demand rate affects the previous analysis in a number of ways.

1. A function must be determined relating demand for documents as a function of time, $\overline{D}(t)$.
2. The probability that a document has demand rate $\lambda(t)$ during week t is reflected in terms of a series of mean demand rates, $\overline{D}(1)$, $\overline{D}(2)$, $\overline{D}(3)$, etc., one for each week.
3. The probability that a copy of a document is on the shelf during a given week will depend upon this variable demand rate, i.e., $P(S(t)|\lambda(t),c)$.

To determine the form of $P(S(t)|\lambda(t),c)$ a queuing model is utilized with a finite number of servers (copies) in which there is no waiting line. This is the situation when we have one or more copies of a book and assume no one places a reserve on a title if a copy is not immediately available when requested.

Let $\lambda(t)$ = rate of demand for a particular document during week t (Poisson arrivals)

$^1/\mu$ = mean time document is out on loan (exponential service time)

$P_n(t)$ = probability there are n copies of a document out on loan during week t

c = number of copies of a particular document in the library system

$$\frac{dP_o(t)}{dt} = -\lambda(t)P_o(t) + \mu P_1(t) \qquad \text{for } n = 0$$

$$\frac{dP_n(t)}{dt} = -\lambda(t)P_n(t) - n\mu P_n(t) + \lambda(t)P_{n-1}(t) + (n+1)\mu P_{n+1}(t) \qquad \text{for } 1 \le n < c$$

$$\frac{dP_c(t)}{dt} = -c\mu P_c(t) + \lambda(t)P_{c-1}(t) \qquad \text{for } n = c \tag{4.14}$$

Since $\lambda(t)$ varies with time, steady state conditions do not exist and this model must be solved for the transient states. Using the generating functions $Z^n$ where

$$\sum_{n=0}^{c} P_n(t)Z^n = P(Z,t) \tag{4.15}$$

we have

$$\frac{\partial P(Z,t)}{\partial t} = \lambda(t)(Z-1)\left[P(Z,t) - Z^c P_c(t)\right] - \mu(Z-1)\,\frac{\partial P(Z,t)}{\partial Z} \tag{4.16}$$

The solution of this linear partial differential equation is quite complex. Indeed, for the case in which there is a single copy of a document, the probability this copy is out at time t, given a demand rate $\lambda(t)$ is as follows:

$$P_1(t) = e^{-\int_0^t(\lambda(x) + \mu)dx}\left[\mu\int_0^t e^{\int_0^t(\lambda(x) + \mu)dx}\,dt + \frac{\overline{D}(0)}{\mu + \overline{D}(0)}\right] \tag{4.17}$$

where we assume demand rate has been constant, $D_o$, prior to $t = 0$.

Since $P(S(t)|c=1, \lambda=\lambda(t)) = 1 - P_1(t)$, evaluation of the integral of this expression times $\lambda P(\lambda)$, to solve for the mean demand rate, $\overline{D}(t)$, would be quite difficult taking the form of:

$$G_i(t) = N_i \int_0^{\infty} \lambda(t)\Big\{1 - e^{-\int_0^t (\lambda(x) + \mu)dx}\Big[\mu\int_0^t e^{\int_0^t (\lambda(x) + \mu)dx}\, dt + \frac{\overline{D}(0)}{\overline{D}(0) + \mu}\Big]\Big\} \frac{1}{\overline{D}_i(t)}\, e^{-\frac{\lambda(t)}{\overline{D}_i(t)}}\, d\lambda(t) \tag{4.18}$$

where

$G_i(t)$ = number of circulations in week t from subject area i

$N_i$ = total number of titles in subject area i

$\lambda(t)$ = demand rate function for circulation of a particular document during week t

$\overline{D}_i(t)$ = mean demand rate during week t for documents in subject area i

$^1/\mu$ = mean loan period in weeks

$\overline{D}(0)$ = demand rate at t = 0.

A demand rate function could be hypothesized for $\lambda(t)$ and the preceding equation could, in theory, be solved for $\overline{D}_i(t)$. This expression is valid only if we assume that these titles have no duplicates. Embellishment of this model to include duplicate copies would make evaluation even more difficult. Because of the difficulty involved in obtaining an exact solution to this model let us explore the effects of a variable demand rate in an attempt to discover a valid approximation.

## Approximate Analysis

Previously we constructed a model assuming that the demand rate for a document is a random variable, $\lambda$, which is constant over time. The model was of the form

$$N_i \int_0^\infty \sum_c \lambda P(S|\lambda,c)P(c|\lambda)P(\lambda)d\lambda \qquad (4.19)$$

with $\lambda$ described by a truncated exponential distribution, $P(\lambda)$, with mean parameter $\overline{D}_i$. If we equate weekly circulation data, $G_i(t)$, to this expression and solve for the weekly mean demand rate, $\overline{D}_i(t)$, we obtain biased results. In general, use of this method will result in overestimates of the true mean demand rate, when the demand rate is increasing over time, and underestimates of the true mean demand rate, when the demand rate is decreasing over time. For as $\overline{D}_i(t)$ is increasing, at any point $t_1$, when the model is integrated with respect to $\lambda$, the function $P(S|\lambda,c)$ is computed as if the mean of $\lambda(t_1)$ is $\overline{D}_i(t_1)$. However, since $\overline{D}_i(t_1)$ is greater than the past demand rate, the function $P(S|\lambda,c)$ yields probabilities that are less than the actual existing probability that a copy is available at time $t_1$. Hence, estimates of $\overline{D}_i(t)$ computed in this manner are greater than the true values of $\overline{D}_i(t)$.

Similarly, when $\overline{D}_i(t)$ is decreasing, if the model is integrated with respect to $\lambda$, the function $P(S|\lambda,c)$ yields values, at any point $t_2$, as a function of $\lambda(t_2)$ with mean $\overline{D}_i(t_2)$. As $\overline{D}_i(t_2)$ is less than the past demand rates, the function $P(S|\lambda,c)$ yields values that are greater than the actual existing probability that a copy is available at time $t_2$. Hence, values of $\overline{D}_i(t)$ computed in this manner are less than the actual values of $\overline{D}_i(t)$. Using this method we obtain overestimates of $\overline{D}_i(t)$ during one portion of the semester and underestimates of $\overline{D}_i(t)$ during the other portion of the semester. For decision-making and planning purposes, it is desirable to estimate the mean demand rate and to evaluate various new title and multiple copy selection policies on a semester or yearly basis. Therefore, aggregation of the weekly $\overline{D}_i(t)$ values over such periods may very well yield sufficiently accurate estimates for the desired $\overline{D}_i(t)$ value. The biases in the high direction will tend to compensate biases in the low direction when these values are aggregated over a semester or year.

An alternative means for estimating $\overline{D}_i(t)$ is to utilize the variable $L_i(t)$ which is a measure of the number of documents out of the library on loan at any time t. Values of $L_i(t)$ can be estimated as a function of $G_i(t)$, the number of weekly circulations, and the length of time documents are borrowed, or can be measured directly by counting the total charge-outs at the end of each week.

If we define

$\ell$ = number of copies of a given title currently out on loan,

$P(\ell|\lambda,c)$ = probability that $\ell$ copies of a title are out on loan given demand rate, $\lambda$, and number of copies, $c$,

and, as before,

$N_i$ = number of titles in the subject area i,

$P(c|\lambda)$ = probability that a title has c copies given that it has demand rate, $\lambda$

then we can formulate a model relating these variables to the number of documents currently out on loan, $L_i(t)$. In this case

$$L_i(t) = N_i \int_0^{\infty} \left[ \sum_c P(c|\lambda) \sum_{\ell=0}^{c} \ell P(\ell|\lambda,c) \right] P(\lambda) d\lambda \qquad (4.20)$$

In this model the number of documents currently out on loan, $L_i(t)$, equals the expected number of copies of a particular title which are currently out on loan, $\sum_{\ell=0}^{c} \ell P(\ell|\lambda,c)$, expected over the conditional distribution of copies given demand rate $\sum_c P(c|\lambda)$, which in turn is expected over the distribution of the demand rate of the title, $P(\lambda)d\lambda$, and finally multiplied by the total number of titles in the subject area, $N_i$. This expression can then be solved for $\overline{D}_i(t)$ which is the mean demand rate for titles in the subject area and the parameter of the $P(\lambda)$ distribution.

As before we will assume that $P(\lambda)$ is described by a truncated exponential distribution and $P(c|\lambda)$ takes on the form of the distribution discussed previously. The probabilities for $P(\ell|\lambda,c)$ are computed from a queuing model assuming a constant demand rate for titles and a situation in which no reserves are placed for titles not immediately available. In this case $P_\ell$, the probability that $\ell$ copies of a title are out on loan, is expressed as:

$$P_\ell = \frac{1}{\ell!}\left(\frac{\lambda}{\mu}\right)^{\ell}\left(\frac{1}{\sum_{\ell=0}^{c}\left(\frac{\lambda}{\mu}\right)^{\ell} \Big/ \ell!}\right) \tag{4.21}$$

Hence,

for c = 1; $$P_1 = \frac{\lambda/\mu}{1 + \lambda/\mu}$$

for c = 2;
$$P_1 = \frac{\lambda/\mu}{1 + \lambda/\mu + \lambda^2/2\mu^2}$$

$$P_2 = \frac{\lambda^2/2\mu^2}{1 + \lambda/\mu + \lambda^2/2\mu^2} \qquad (4.22)$$

for c = 3; etc.

Solution of this model yields values which underestimate the true mean demand rate when $\overline{D}_i(t)$ is increasing over time and overestimate the true mean demand rate when $\overline{D}_i(t)$ is decreasing over time. The reason for this error in estimation is that the function $P(\ell|\lambda,c)$ in the model, at any point $t_1$, yields values based upon the assumption that the past demand rate has been constant, equal to $\overline{D}_i(t_1)$. Since $\overline{D}_i(t)$ is increasing over time, at any point $t_1$, the value yielded by $P(\ell|\lambda,c)$ is greater than the actual probability that $\ell$ copies of a title are out on loan. If the equality in the model is to hold, the solution must yield a value of $\overline{D}_i(t_1)$ which is less than the true demand rate at time $t_1$. Similarly when $\overline{D}_1(t)$ is decreasing over time, at any point $t_2$, the function $P(\ell|\lambda,c)$ will yield values which are less than the true probability that $\ell$ copies of a title are out on loan. Solution of the model will then yield a value for $\overline{D}_i(t_2)$ which is greater than the actual demand rate at time $t_2$.

By employing both models, two estimates for $\overline{D}_i(t)$ are obtained, one of which is an overestimate and one of which is an underestimate. This analysis provides a method for bracketing the mean demand rate. An analysis can then be performed to determine the sensitivity of the estimates to various functions of $\overline{D}_i(t)$ and a judgment can be rendered concerning the accuracy of the demand rate estimates yielded by the circulation model when aggregated over a semester or year when the demand rate varies over time. These two methods of estimation are displayed schematically in Figure 4.4. In this figure, method #1 refers to the solution of

$$G(t) = N \int_0^\infty \lambda \sum_c P(S|\lambda,c)P(c|\lambda)P(\lambda)d\lambda \qquad (4.23)$$

for each week t, yielding estimates of the mean demand rate, $\overline{D}(t)$. Method #2 refers to the solution of

$$L(t) = N \int_0^\infty \left[\sum_c P(c|\lambda) \sum_{\ell=0}^{c} \ell P(\ell|\lambda,c)\right] P(\lambda)d\lambda \qquad (4.24)$$

for each week t, yielding estimates of the mean demand rate, $\overline{D}(t)$.

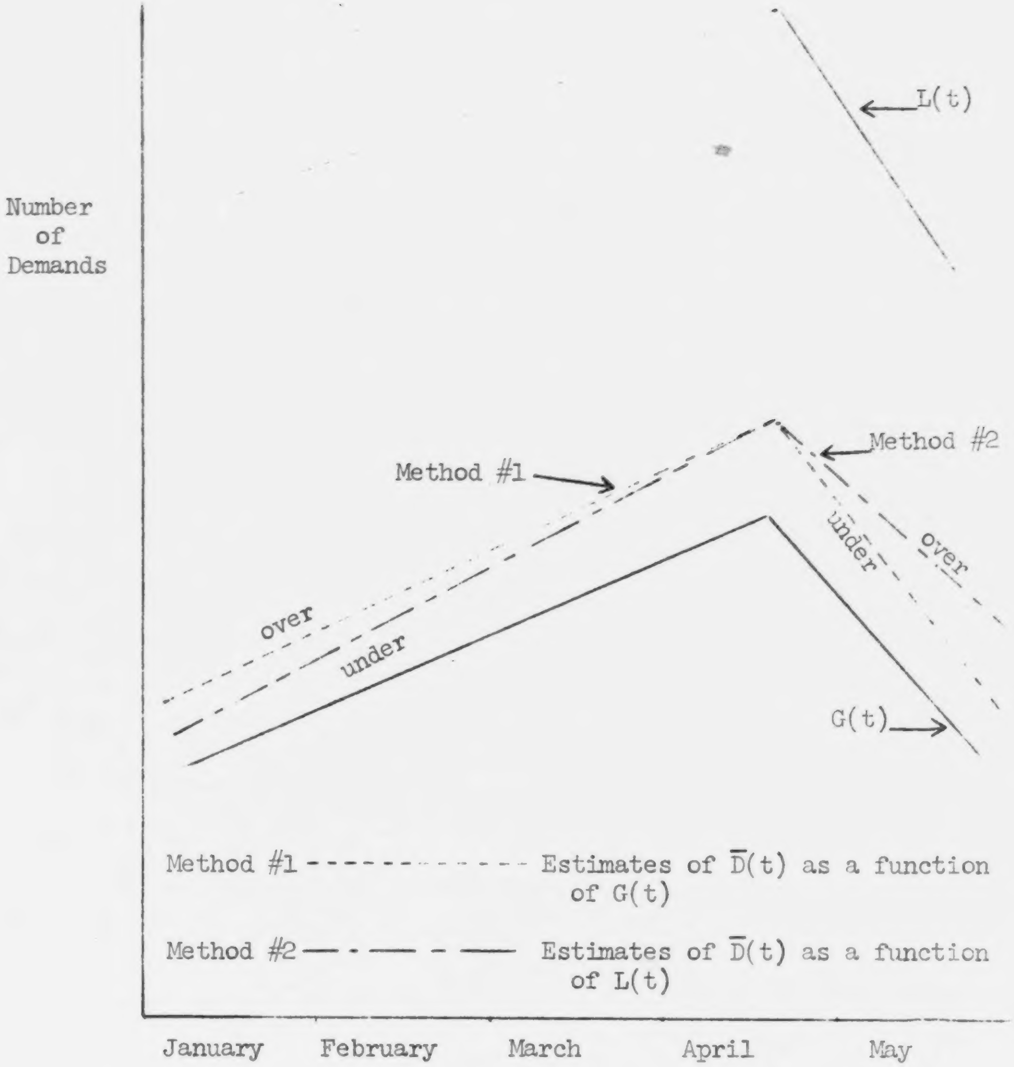

Fig. 4.4--Estimates of the Weekly Demand for Documents as a Function of Weekly Circulations, G(t), and Number of Documents Out on Loan Per Week, L(t).

## Conclusions

The preceding discussion has centered on a model which can be used to estimate the demand rate for the circulation use of titles within a given subject area as a function of the circulation rate and/or the number of documents out on loan at a particular point in time. In addition, this model provides a means for predicting the mean demand rate for titles in a subject area as function of the research and teaching activity related to the area making it possible to predict future demands for circulation use as the structure and composition of the university change. This model considers the case in which there are multiple copies of a title. In addition, an approximate estimate of the mean demand rate is rendered in situations in which the demand rate fluctuates over time. Finally, except for some initial sampling of titles to obtain past circulation use to verify the model and estimate a few parameters, circulation data necessary to implement the model are readily obtainable as a by-product of the circulation service administrative process.

## Demand for In-Library Use of Documents Which are Allowed to Circulate

In this section a model will be constructed to predict demand for in-library use of documents permitted to circulate. The model presented here relates in-library book use with circulation use on a per subject area basis. Fussler and Simon found that in general "books that develop little recorded use develop little browsing, and books that develop much recorded use develop much browsing, except for the highest used books (which develop little browsing as they are usually not available for browsing as they are out on loan)..."[11]

In another study McGrath[12] found a rather high positive correlation ($r = .86$) between books circulated and books used in the library for a group of 61 different subjects. Hence, an approach which relates browsing use to actual recorded circulation use would seem reasonable.

Therefore, if we define $P(S|\lambda,c)$, $P(c|\lambda)$, $P(\lambda)$, $N_i$ and $G_i$ as in the previous section and let

$H_i$ = number of documents consulted in the library in subject area i during a specified time period (e.g., a week)

$\beta$ = a coefficient relating in-library use to circulation use

then, the probability that at least one copy of the title, with demand rate $\lambda$, is on the shelf is

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[11] Fussler and Simon, Patterns in The Use of Books, p.115.

[12] William McGrath, "Correlating the Subjects of Books Taken Out of and Books Used Within an Open-Stack Library," College and Research Libraries, 32:280-285 (July 1971).

$$P(S|\lambda) = \sum_c P(S|\lambda,c)P(c|\lambda) \tag{4.25}$$

and the demand rate for in-library use of a particular title is $\beta\lambda$. Therefore, the expression for the number of books used in subject area i during a specific period is as follows:

$$H_i = N_i \int_0^\infty \beta\lambda P(S|\lambda)P(\lambda)d\lambda$$

and (4.26)

$$H_i = N_i \int_0^\infty \beta\lambda \sum_c P(S|\lambda,c)P(c|\lambda)P(\lambda)d\lambda$$

This expression can be solved in a direct fashion as follows:

$$\beta = \frac{H_i}{N_i \int_0^\infty \lambda \sum_c P(S|\lambda,c)P(c|\lambda)P(\lambda)d\lambda} \tag{4.27}$$

But we note that the denominator is actually equal to $G_i$, the number of circulations in subject area i.

Hence, $$\beta = \frac{H_i}{G_i} \tag{4.28}$$

Therefore, the predicted number of actual in-library uses is $\beta G_i$ and the predicted number of demands for in-library use can be expressed as $\beta D_i$ where $D_i$ is the predicted number of demands for circulation use in subject area i during a given period. Here the assumption is made that if in-library use can be estimated as a linear function of circulation use, then the demand for in-library use can be estimated as a linear function of the demand rate for circulation use. There is no reason to believe that this assumption will contribute a significant degree of bias.

One of the most comprehensive studies of in-library book use was conducted by Fussler and Simon.[13] The research technique employed in their study was to place a questionnaire in each of a sample of documents in the physics and history collections. When a user removed the document from the shelf containing a questionnaire, he was requested to complete and turn in the questionnaire. Another technique, employed successfully by Elston,[14] consists of counting for each subject area the number of documents which are left out on tables and desks by users. As some users reshelve documents after use, contrary to the desires of most library administrators, this effect would have to be estimated. This is analogous to the problem encountered by

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[13] Fussler and Simon, Patterns in the Use of Books, p.116.

[14] Caroline Elston, "Survey of In-Library Use in M.I.T. Science Library," O.R. Course 8.75 Term Report, M.I.T. Libraries, Cambridge, Mass., 1966, p.2.

Fussler and Simon when users replaced questionnaires in documents without completing them. Utilization of this counting technique provides an arbitrary definition for consulting a book in the library. A document is consulted in the library if a user removes it from the shelf and carries it to a study area. Utilization of this technique provides a simple and expedient means for data collection requiring little cooperation from users. In this case users would merely be asked not to reshelve documents and stack personnel would note call numbers for documents which they reshelved.

Other studies performed in this area have employed the technique of interviewing users[15] and still other studies have made use of the diary technique.[16] In each of these techniques there is an inherent degree of bias when users are asked to provide information regarding their behavior in the library. Specifically, users are likely to behave differently than they usually do or to provide information inconsistent with actual behavior when keeping a diary or responding to an interviewer. In addition, each of these techniques is time consuming and expensive to administer.

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[15] Saul Herner, "A Pilot Study of the Use of the Stacks of the Library of Congress," (unpublished, Washington, D. C., Herner and Co., 1960).

[16] Alice Bowen, "Nonrecorded Use of Books and Browsing in the Stacks of a Research Library," (M.A. Thesis, The University of Chicago Graduate Library School, 1961).

We can see that there are a number of techniques available to collect data on in-library use of books. Some auxiliary data are necessary to utilize any of these techniques. The use of the technique of counting documents left on tables requires an estimate of the proportion returned to the stacks by users. The use of the questionnaire technique requires an estimate of questionnaires ignored by users. The diary and interview techniques require knowledge of the number of users of the library and an estimate of the possible bias.

Since we would expect the demand rate for in-library use of books to vary over a semester, a problem arises similar to that encountered when estimating the demand rate for the circulation use of books. However, since the demand for in-library use is assumed to be a linear function of the demand for circulation use, this problem is resolved when the corresponding problem is resolved for estimating the demand rate for circulation use.

In this section a technique is proposed to estimate the demand rate for in-library use of titles as a function of the demand rate for circulation use of titles in each subject area. An advantage of utilizing this technique is that no additional data collection effort is required except for periodic sampling to verify the estimate of the parameter, $\beta$. Given an estimate of $\beta$, the expected number of in-library uses is estimated to be $(\beta)(G_i)$, whereas the expected number of demands for in-library use is estimated to be $(\beta)(D_i)$ for documents in subject area i.

## Multiple Copy Selection Model

In a preceding section, a Circulation Demand Model was developed to estimate the mean demand rate, $\overline{D}_i$, for the circulation use of titles held in subject area i as a function of the recorded number of circulations, $G_i$, in that subject area. In addition, a technique was developed for estimating the demand rate for in-library use of titles as a function of the demand rate for circulation use of titles. The output of this model was then used to construct a model which can be used for predicting mean demand rate as a function of variables descriptive of the teaching and research efforts of the university. In this section it is demonstrated how the output of these models can be used in the decision process to provide guidance in the selection of multiple copies for titles. Specifically, a model is developed which estimates the expected number of additional circulation uses experienced by the purchase of an additional copy of a title. Measures of exposures per unit cost are generated for various multiple copy purchasing strategies which can then be used to determine an effective strategy for the purchase of multiple copies.

## Subcollection Demand Rate Estimation

Up to this point the collection of books held within a particular subject area has been viewed as an entity of a specific size. In reality the size of a collection is continually changing with new books being added and old books being weeded. In general, purchasing decisions focus on those books published in the most recent year, whereas weeding decisions focus on books published in prior years. It would seem that a measure of the number of titles purchased or the proportion of titles published each year which are purchased would be indicative of the library's ability to meet a great diversity of demands. On the other hand, the proportion of titles for which duplicate, triplicate, etc. copies are purchased would seem to be a measure of a library's ability to meet the relatively high intensity of demands experienced by a portion of the titles in the collection. Hence, a meaningful decision variable governing the purchase of multiple copies would seem to be the proportion of new titles for which duplicate, triplicate, etc. copies are purchased.

Each collection can be viewed as a series of subcollections with a subcollection comprised of those books published in a particular year. Corresponding to each subcollection is a probabilty distribution descriptive of the demand rate for each title in the subcollection. The mean demand rate for titles within subject area i which are t years old is denoted by $\bar{D}_{it}$ where t = current year minus the publication date of the title. We would expect that the mean demand rate for titles in a subcollection tends to decrease as t increases. This decrease would be observed as a result of the obsolescence of knowledge with age. This expected phenomenon is depicted in Figure 4.5.

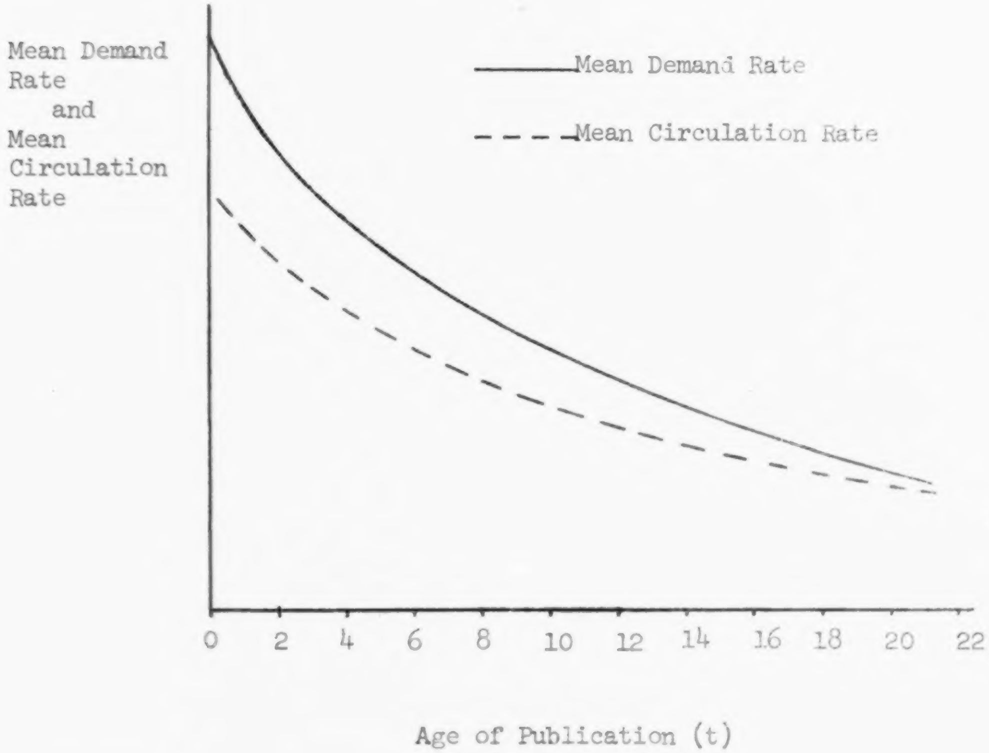

Fig. 4.5--Mean Demand Rate and Mean Circulation Rate Per Title vs. Age of Titles in Subject Area i.

We would expect the mean circulation rate per title in a subcollection, as defined by the t, to behave in a similar fashion. However, we would expect that values of the mean circulation rate would be somewhat less than the values of the mean demand rate, especially for high values of mean demand rate. This difference is indicative of the unsatisfied demands resulting when users request titles which are out on loan.

If we let

$N_i$ = number of titles held in subject area i

$\overline{D}_i$ = mean demand rate for titles in subject area i

$N_{it}$ = number of titles t years old held in subject area i

$\overline{D}_{it}$ = mean demand rate for titles in subject area i of age t

then the mean demand rate for titles in subcollections relates to the mean demand rate for the entire collection of subject area i in the following manner.

$$\sum_t \overline{D}_{it} N_{it} = \overline{D}_i N_i \tag{4.29}$$

To obtain estimates of the mean demand rate for titles in each subcollection, a technique which yields approximate estimates is employed. To utilize this method, circulation data must be collected and recorded according to the publication date and subject area for each title borrowed. In this manner an estimate is obtained of the number of documents which are borrowed from subject area i, subcollection t, $G_{it}$, during a specified period of time. Then $G_{it}/G_i$ indicates the ratio of the number of documents which are borrowed from subject area i, subcollection t, to all documents which are borrowed from subject area i during a specific period of time. Estimates of the number of documents held in subject area i, subcollection t, $N_{it}$, must also be obtained. Then, $N_{it}/N_i$ represents the ratio of the number of documents in subject area i, subcollection t, to all documents held in subject area i. The ensuing ratio of these ratios, $\rho_{it}$, represents the relative circulation use rate experienced by documents in subject area i, subcollection t where

$$\rho_{it} = \frac{G_{it}/G_i}{N_{it}/N_i} \tag{4.30}$$

Thus given a vector of values for $\rho_{it}$, an estimate of the circulation use rate experienced by the documents in subject area i, subcollection t, is equal to

$$\overline{G}_{it} = \rho_{it}\overline{G}_i \tag{4.31}$$

To obtain estimates of $\rho_{it}$ values for each subcollection of each subject area requires extensive data collection efforts, especially if the circulation service system is not automated. However, since we would not expect the vector of $\rho_{it}$ values to vary significantly from one subject area to another within a broad subject category a single vector of $\rho_t$ based upon circulation and holdings data for the combined subject areas can be computed. The use of this combined estimate for $\rho_t$ probably does not introduce a significant degree of error.

Values of the demand rate for circulation use of titles in subject area i, subcollection t, are estimated in a similar fashion. Here, an approximate estimate of the mean demand rate for titles in subject area i, subcollection t, is computed as

$$\overline{D}_{it} = \rho_t \overline{D}_i \tag{4.32}$$

This represents an approximate estimate of $\overline{D}_{it}$ as $\rho_t$ was computed based upon circulation rate data. Since the demand rate and circulation rate increase disproportionately with respect to each other, the use of $\rho_t$, based upon circulation rate data, to apportion demand rate, will result in a slight degree of bias. The degree of bias can be determined by obtaining an estimate of $\rho_t$ based upon demand rate data. This could be accomplished by solving the Circulation Demand Model to determine estimates of $\overline{D}_{it}$ for each subcollection t. The results of using this vector of $\rho_t$ values computed on the basis of these demand rate estimates can then be compared to the results obtained using the vector of $\rho_t$ values based upon circulation rate data.

## Assumptions

Before proceeding further in the development of this model certain assumptions are considered. One assumption centers upon the use of cross-sectional data to estimate time series parameters. Ideally it is desirable to collect data of the number of circulations experienced by a subcollection of books from year to year as the subcollection ages (time series data). From these data a model could be constructed and parameters estimated to predict the additional number of circulation uses experienced as a result of different multiple copy selection strategies over the life of these books. These data are generally unavailable. In order to estimate these parameters, we use data for all of the subcollections for a single period of time (cross-sectional data). These data are then used in lieu of data descriptive of a single subcollection over time. Hence, the assumption involved in the cross-sectional analysis is that the variables relevant to this analysis behave in similar fashion for one subcollection transcending time as for a series of subcollections during one period in time.

Another assumption made is that titles which are initially most popular remain the most popular as the titles of the subcollection age. In general, we would expect that the titles in a subcollection, which experience the greatest demand rate when first purchased will continue to experience the greatest demand rate as compared to other titles in the subcollection as the subcollection ages. Another assumption made is that titles which experience the greatest demand rate are those titles for which multiple copies are purchased. Obviously, no perfect selection mechanism exists to insure that this will be the case. However, a combination of some judicial decision making on the part of the selectors, aided by faculty recommendations and attention to those titles which receive a number of reserve requests, should insure that a high proportion of titles receiving the highest demand rate will have multiple copies. Finally, it is assumed that the demand rate for titles within the various subcollections is described by a truncated exponential distribution with mean demand rate $\overline{D}_{it}$ and a discrete non-zero probability at $\lambda = 0$. This assumption is similar in nature to the assumption made for the entire collection in the Circulation Demand Model.

An implicit assumption employed in this analysis is that an exposure experienced by a multiple copy of a title in subject area i has the same value as an exposure experienced by a multiple copy in subject area j. This assumption is not necessary for this analysis. If it is desirable, the exposures experienced by the multiple copies of titles in different subject areas can be assigned weights as discussed in Chapter II. In fact, if it is desirable, different weights can be assigned to exposures experienced by the duplicate, triplicate, etc. copy.

In this analysis it is assumed that a demand for a title was permanently lost if a copy of the title was not immediately available. This we know is not necessarily valid. Little is known concerning user behavior if a demand is not immediately satisfied. Morse estimates that a reserve is placed on a title when a copy is not immediately available one time in eight.[17] He does not consider the probability of the event that a user might return at a later date if a copy of a title is not immediately available. On the other hand we know that only a portion of the users who place a reserve on a title do return to the library to circulate the title when notified that it is available. In addition, a user returning to the stacks at a later date encounters a fair probability that a copy of the title is still unavailable. Undoubtedly these user activities of placing a reserve and eventually withdrawing a copy of the title and returning at a later date and finding a copy of the desired title available, have the effect of reducing the estimate of the expected number of lost exposures. However, we tend to think that the magnitude of the reduction is not very substantial. User behavior in this area is certainly deserving of further research.

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[17] Morse, Library Effectiveness, p.145

Duplicate Copy

With these assumptions let us now consider the purchase of the second or duplicate copy of a title. We let m = the proportion of titles for which duplicate copies are purchased. Let us further assume that the titles for which duplicate copies are purchased are included in the set of the m' proportion of titles which experience the higher demand rate within the subcollection, where $m \leq m'$. The implication made here is that it is possible to divide new titles into two groups: those that will receive a relatively high demand rate; and those that will not. Of the titles in the set expected to receive the higher demand rate, duplicates are purchased for $^{m}/m'$ proportion of these. The set of m' proportion of titles, for which $^{m}/m'$ proportion are duplicated, is represented by the shaded area in Figure 4.6.

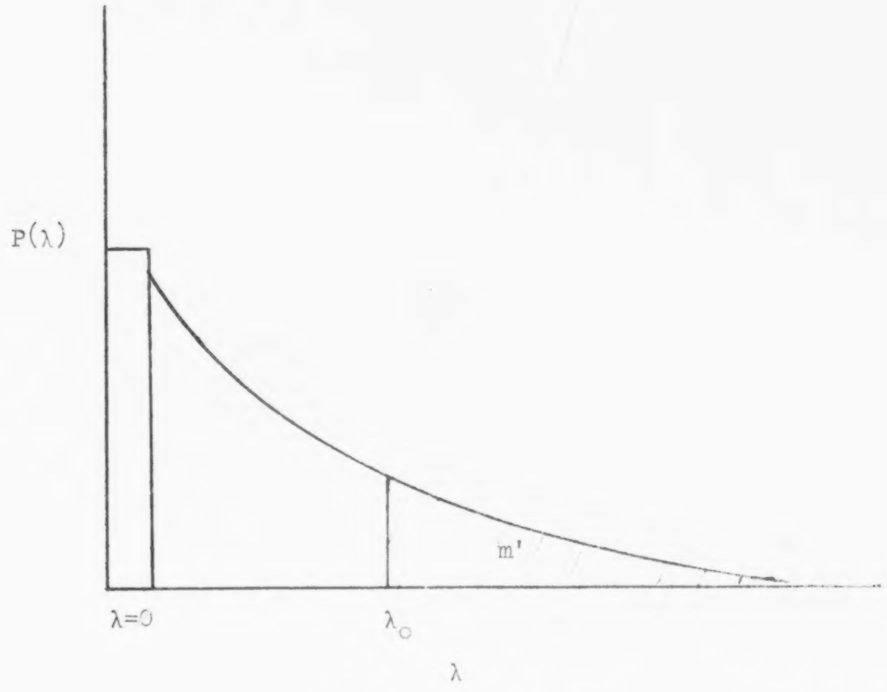

Fig. 4.6-- Probability Distribution of the Demand Rate for Titles, λ, Which are t Years Old With Shaded Area Representing the m' Proportion of Titles Which Receive the Higher Demand Rate.

$\lambda_o$, the demand rate which partitions the set of m' proportion of titles which experience the higher demand rate from the set of 1 - m' proportion of titles which receive a lower demand rate, is determined as follows:

$$\int_{\lambda_o}^{\infty} \frac{1}{\overline{D}t} e^{-\lambda/\overline{D}_t} d\lambda = m'$$

$$\therefore \lambda_o = -\overline{D}_t \ln m' \qquad (4.33)$$

If a library employs a perfect decision mechanism in determining which titles were to be duplicated, then $\lambda_o = -\overline{D}_{it} \ln m$ , as $m' = m$. This is the case in which the set of m proportion of titles duplicated is the same set which receive a higher demand rate. If we assume that the library uses no system, e.g., a set of m proportion of titles selected at random are duplicated, then $\lambda_o = 0$ as $m' = 1.0$ and $\ln 1.0 = 0$. These two cases represent the extreme conditions. There are an infinite number of in-between cases relating to the two variables of

1. the proportion of titles for which duplicate copies are purchased (m), and
2. the relative ability of the library in selecting duplicate copies for those titles which experience a higher demand rate ($^{m}/m'$).

Finally, it is assumed that the set of $^{m}/m'$ proportion of titles duplicated for the set of m' titles which receive a higher demand rate are duplicated at random within this set. This assumption, although somewhat arbitrary, should not significantly affect the results.

Undoubtedly, decision mechanisms other than the one presented in this dissertation (duplicate the set of $^m/_{m'}$ proportion of the set of m' proportion of titles expected to receive a higher demand rate) can be evolved. However, we believe that the one developed in this research is reasonable. In addition, the form of this decision mechanism makes it tractable for inclusion into an analytical decision model.

Given that there exists a demand rate $\lambda$ for a particular title, the expected number of circulations experienced by this title if there are two copies, equals $\lambda$ times the probability that at least one copy of the title is on the shelf ready for use. Using the results presented in equation (4.10), this can be represented symbolically by

$$(\lambda)(P(n < 2|\lambda,c=2) = (\lambda)(1 - \frac{\lambda^2/2\mu^2}{1 + \lambda/\mu + \lambda^2/2\mu^2}) \qquad (4.34)$$

where n = the number of copies in use. The expected number of circulations experienced by this same title if there is a single copy is equal to $\lambda$ times the probability that the copy is on the shelf ready for use.

Symbolically, this is represented by

$$(\lambda)(P(n=0|\lambda,c=1)) = (\lambda)(1 - \frac{\lambda/\mu}{1 + \lambda/\mu}) \qquad (4.35)$$

Then, the additional expected number of circulations experienced by purchasing a duplicate copy for this title is equal to the difference between equations 4.34 and 4.35.

$$\lambda\left[1 - \frac{\lambda^2/2\mu^2}{1 + \lambda/\mu + \lambda^2/2\mu^2}\right] - \lambda\left[1 - \frac{\lambda/\mu}{1 + \lambda/\mu}\right]$$

$$= \lambda\left[\frac{\lambda/\mu}{1 + \lambda/\mu} - \frac{\lambda^2/2\mu^2}{1 + \lambda/\mu + \lambda^2/2\mu^2}\right] \qquad (4.36)$$

Then, assuming that the set of $^m/m'$ proportion of titles are duplicated of those titles which experience a demand rate of $\lambda_o$ or more, the expected number of circulations experienced by the duplicate copy for a title of age t is:

$$M_t = \frac{1}{\overline{D}_t} \int_{\lambda_o}^{\infty} \lambda\left[\frac{\lambda/\mu}{1 + \lambda/\mu} - \frac{\lambda^2/2\mu^2}{1 + \lambda/\mu + \lambda^2/2\mu^2}\right] e^{-\lambda/\overline{D}_t} d\lambda \qquad (4.37)$$

where $M_t$ = expected number of circulations experienced by the duplicate copy of a title in a subcollection of age t

$\overline{D}_t$ = mean demand rate for titles which are of age t

$^1/_\mu$ = mean loan period of titles

m' = proportion of titles which experience a demand rate of $\lambda_o$ or more

$\lambda_o = -\overline{D}_t \ln m'$

The expected number of circulations experienced by a duplicate copy over the life of the title is $M = \sum_{t=1}^{\infty} M_t$. If we discount exposures over time, as discussed in Chapter II, the expression becomes

$$M = \sum_{t=1}^{\infty} \frac{M_t}{(1 + r)^t} \tag{4.38}$$

Given this expression and the cost figure associated with the purchase of a duplicate copy, a benefit-cost measure can be constructed. We expect that the cost of adding an additional copy to be somewhat less than the cost of adding an additional title. In the former case the cost of classifying and cataloging would be significantly less.

To construct the marginal benefit-cost ratio for use in determining the optimal proportion of titles to duplicate, a measure of the change in the expected number of circulations per unit cost is required as m and m' vary. If we let

$M_i^{m'}$ = the expected number of circulations experienced per duplicate copy in subject area i over the life of the copy when duplicates are purchased for $^m/m'$ proportion of titles in the set of m' titles which will experience a higher demand rate ($\lambda_o$ or more),

then

$(M_i^{m'})(N_{it})(m)$ = the total number of expected circulations experienced by the duplicate copies in subcollection t, in subject area i over the life of the copies when $^m/m'$ proportion of titles are duplicated of those m' titles which receive a higher demand rate ($\lambda_o$ or more),

where $N_{it}$ = the number of titles in subcollection t, subject area i.

If $F_1$ = the average cost associated with procuring an additional copy of a title,

then $(F_1)(N_{it})(m)$ = the cost of procuring m proportion of duplicate copies for the titles in subject area i, subcollection t.

Finally, we let

$W_i$ = the estimated mean number of equivalent exposures (item-use-days) for one circulation use as depicted in Figure 2.2.

The marginal number of expected exposures per unit cost in subject area i, if the proportion of new titles duplicated is increased from $m_1$ to $m_2$, is seen to be

$$\frac{\Delta M_i}{\Delta C} = \frac{(W_i)\left[(M_i^{m_2'})(N_{it})(m_2) - (M_i^{m_1'})(N_{it})(m_1)\right]}{(F_1)(N_{it})(m_2) - (F_1)(N_{it})(m_1)}$$

$$= \frac{(W_i)\left[(M_i^{m_2'})(m_2) - (M_i^{m_1'})(m_1)\right]}{F_1(m_2 - m_1)} \qquad (4.39)$$

Since the number of expected additional exposures gained by adding duplicate copies is a function of m (the proportion of titles for which duplicates are purchased), and m' (the proportion of titles which comprise the set of titles expected to receive a higher demand rate), and since the cost of the duplicate copies is also a function of m, a schedule can be constructed relating expected additional exposures to additional costs as a function of m. This schedule of benefit-cost ratios can then be used to determine the optimal value of m. The optimal value of m is determined when the marginal benefit-cost ratio of purchasing an additional duplicate copy equals the marginal benefit-cost ratio for the expenditure of these additional funds in other areas.

Triplicate Copy

In a similar fashion this analysis can be extended for the purchase of a third copy of a title. Here we let

$Q_t$ = the expected number of exposures experienced by the triplicate copy of title in sub-collection t

q = the proportion of titles for which triplicate copies are purchased

q' = the proportion of titles which experience a higher demand rate

Here we assume a similar decision mechanism in which new titles can be divided into a set of titles which experience a higher demand rate and a set of titles which experience a lesser demand rate. We then assume that triplicate copies are purchased for a set of $^{q}/q'$ proportion of titles of the set of q' proportion of titles which receive this higher demand rate. In general, we assume that the value of q' for purchasing triplicate copies is less than the value of m' for purchasing duplicate copies. Then, it follows that triplicate copies are purchased for the set of $^q/q'$ proportion of the titles exhibiting a demand rate greater than $\lambda_1 = -\bar{D}_t \ln q'$.

Given a demand rate $\lambda$, and that there are three copies of a title, the number of circulations experienced by this title is $\lambda$ times the probability that at least one copy is available for circulation use. Symbolically, the number of circulations if there are three copies, is

$$(\lambda)P(n < 3|\lambda,c=3) = (\lambda)(1 - \frac{\lambda^3/6\mu^3}{1 + \lambda/\mu + \lambda^2/2\mu^2 + \lambda^3/6\lambda^3}) \qquad (4.40)$$

If there are two copies of the title, the expected number of circulations experienced by this title is $\lambda$ times the probability that at least one copy is on the shelf ready for use. This can be expressed symbolically by

$$(\lambda)(P(n < 2|\lambda,c=2) = \lambda(1 - \frac{\lambda^2/2\mu^2}{1 + \lambda/\mu + \lambda^2/2\mu^2}) \qquad (4.41)$$

The additional number of circulations expected when the third copy is added equals the difference between equation (4.40) and (4.41).

$$\lambda\left[\frac{\lambda^2/2\mu^2}{1+\lambda/\mu+\lambda^2/2\mu^2}-\frac{\lambda^3/6\mu^3}{1+\lambda/\mu+\lambda^2/2\mu^2+\lambda^3/6\mu^3}\right] \qquad (4.42)$$

The expected number of circulations experienced by each triplicate copy in the set of titles of age t for which triplicate copies are purchased is:

$$Q_t=\frac{1}{\overline{D}_t}\int_{\lambda_1}^{\infty}\lambda\left[\frac{\lambda^2/2\mu^2}{1+\lambda/\mu+\lambda^2/2\mu^2}-\frac{\lambda^3/6\mu^3}{1+\lambda/\mu+\lambda^2/2\mu^2+\lambda^3/6\mu^3}\right] \cdot e^{-\lambda/\overline{D}_t}d\lambda \qquad (4.43)$$

where $\lambda_1 = \overline{D}_t \ln q'$.

The remainder of the analysis proceeds in the same manner as that for the addition of a duplicate copy. The final result is a schedule of benefit-cost ratios which indicates the marginal number of exposures per unit cost for various policies of purchasing triplicate copies for q proportion of the new titles. In a similar fashion this analysis can be extended for the purchase of the fourth, fifth, etc. copy of a title.

## Conclusion

This concludes the development of a model to aid library decision makers in determining effective strategies for the purchase of multiple copies. The output of this model yields estimates of the expected number of circulations experienced by the purchase of an additional copy for various values of the decision variable (the proportion of titles for which an additional copy is purchased). In the next section a model is developed to aid the library decision maker in evaluating different strategies for the purchase of additional new titles.

## Title Selection Model

In a preceding section a model was developed for predicting the number of demands as a function of the actual number of circulations of documents held by the library. At any point in time the difference between these two variables represents the number of unsatisfied requests for documents held by the library. Demands for documents not held by the library (i.e., not part of the library's collection) were not considered in this analysis. In this section a model is developed to predict the additional number of exposures, $\Delta E$, expected because of the purchase of one additional new title. We define a new title as one which was published during the past calendar year. The output of this model serves to guide the administrator in determining the optimal number of new titles to purchase each year in each subject area.

Relevant Variables and Relationships

In constructing the model, we assume that the expected value of $\Delta E$ yielded as the result of purchasing one new additional title in a particular subject area is a function of

1. the intensity of the teaching and research efforts related to the subject area, and
2. the number of titles purchased of the titles published during the past calendar year.

Undoubtedly other variables affect the number of exposures experienced by an additional new title. However, efforts were made to construct a simple model, utilizing variables for which data could be easily collected while still obtaining the desired output.

An example of one variable excluded from the model is a measure of the ratio of titles purchased to the titles published. We assume that the library places the highest priority on the purchase of those titles which are expected to experience the highest demand rates. Then, as the ratio of new titles purchased to the new titles published approaches 1.0, we predict that a measure of the expected number of exposures per title decreases. Unfortunately, collection of data relevant to the number of new titles published each year is an extremely difficult task. No central catalog exists which contains a complete listing of the new titles published according to publication year and subject area. One catalog lists titles, submitted by American publishers to be cataloged by the Library of Congress, according to the Dewey Decimal Classification System.[18] Not included in this catalog are many foreign publications and numerous domestic publications not published by a publishing firm and which receive a limited distribution, e.g., reports, studies, etc. In addition, unless the library is classifying documents using the Dewey Decimal Classification System, it is a considerable task to align subject areas with the appropriate Dewey Decimal call numbers. The magnitude of this task increases when we consider that the Dewey System has undergone a number of major changes in the past few years. Finally, some subject areas contain a large number of publications which would never be considered for purchase by a university library while others contain very few publications which would not be considered relevant to an academic library. Thus, considering these difficulties, reliable estimates of the number of relevant domestic publications can be obtained only through an item by item perusal of the titles in the catalog. Even this effort would not include a count of the numerous special reports, studies, etc. published each year, nor would it include a count of foreign publications. Considering the enormity of the effort required to obtain data for the number of titles published each year in each subject area, a measure of the ratio of new titles purchased to those published is not considered a feasible variable for this model.

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[18] R. R. Bowker Co., The American Book Publishing Record, R. R. Bowker Co., New York

In general we expect that the number of demands satisfied for the titles published during a given year increases, at a decreasing rate, as the number of these titles held in the subcollection by the library increases. We would expect this function to appear somewhat as depicted in Figure 4.7

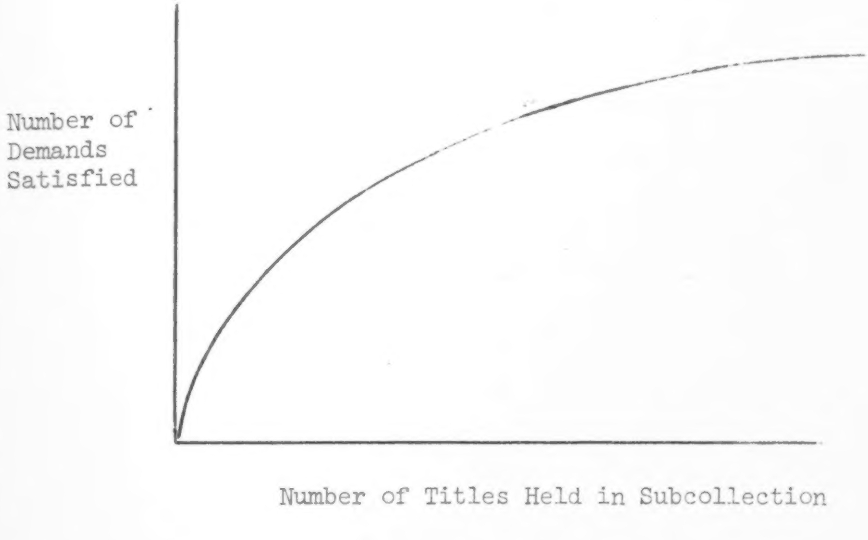

Fig. 4.7--Relationship Between Number of Demands Satisfied and Number of Titles Held in a Subcollection.

We would expect this type of relationship to occur as a result of librarians placing the highest purchase priority on those titles which are expected to receive the highest demand rate. Then, as titles with a lesser expected demand rate are purchased, we would expect the number of demands to rise at a slower rate. Finally, a point is reached at which all possible titles are purchased or the addition of a new title adds virtually zero expected additional exposures. Thus, given that

$E_{it}$ = expected number of exposures by titles in subject area i, subcollection t, during current year, and

$N_{it}$ = number of titles held in subject area i, subcollection t,

one possible function describing this relationship, where $\alpha$ and $\theta$ are parameters, is

$$E_{it} \sim \alpha N_{it}^{\theta} \tag{4.44}$$

No implication is made concerning cause and effect in this relationship. On one hand the greater the number of titles held, the greater the number of demands satisfied. On the other hand the greater the number of titles held, the greater the number of demands created. Hence a collection of titles can be viewed in a passive sense, as satisfying user demands, and/or in an active sense, as stimulating or generating user demands.

Another factor which needs to be incorporated into the model describes the obsolescence of knowledge effect which occurs as titles age. In general we would expect that the demand rate for a set of titles published in a particular year decreases as the set of books age. Elston[19] found that an exponential function could be used as a reliable estimator of this phenomenon for the use of periodicals as they age. We might expect that a similar function can be used to describe the obsolescence of knowledge effect experienced by titles as depicted in Figure 4.8.

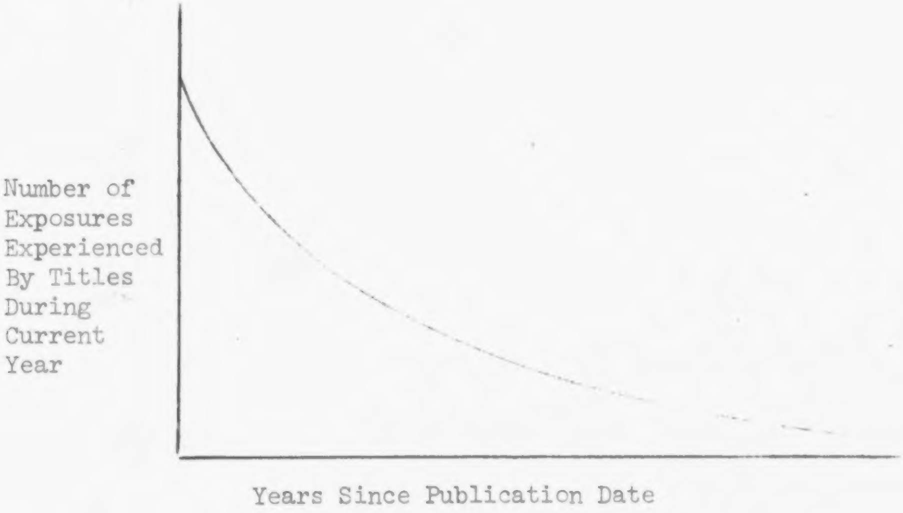

Fig. 4.8--Number of Exposures Experienced by Titles During Current Year for Titles Published in Year t, Where t = 0 in Current Year.

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[19] Elston, "Survey of In-Library Use in M.I.T. Science Library," p.4.

Incorporation of this factor into the model is accomplished as follows:

$$E_{it} \sim \alpha\, e^{-\gamma t} N_{it}^{\theta} \qquad (4.45)$$

where $e^{-\gamma t}$ = factor descriptive of obsolescence of knowledge for titles in subcollection t where $\gamma$ is a parameter.

Finally, we would expect that the number of exposures increases as the size of the university community with interests in a given subject area increases. This relationship might look somewhat as depicted in Figure 4.9.

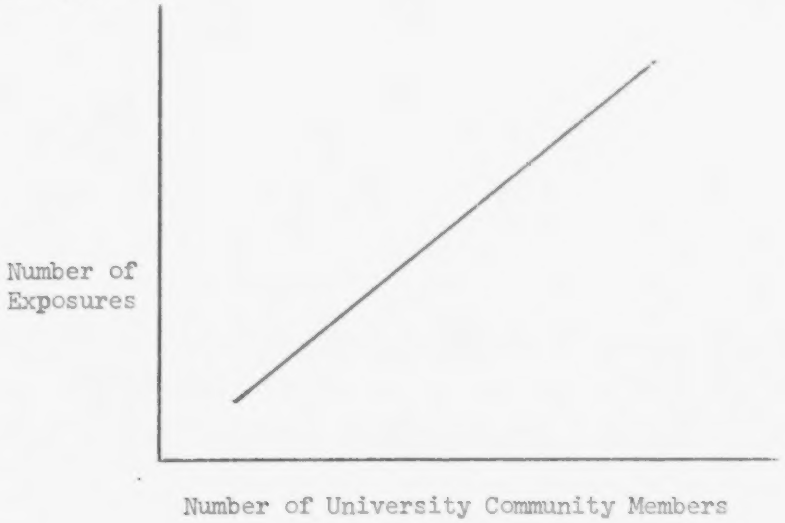

Fig. 4.9--Relationship Between Number of Exposures and Number of University Community Members With Interests in a Particular Subject Area.

One form of the model relating these variables might be

$$E_{it} \sim \alpha e^{-\gamma t} N_{it}^{\theta}(X_{1i} + \delta X_{2i})^{\eta} \qquad (4.46)$$

where $X_{1i}$ and $X_{2i}$ = measures of the number of university community members with interests in subject area i,

and where $\delta$ and $\eta$ are parameters.

We observe that for values of $\alpha$, $\gamma$, $\theta$, $\delta$, and $\eta$ greater than zero

1. the expected number of exposures increases in response to an increase in university membership indicating that, to some extent, the library exists to satisfy user demands
2. the expected number of exposures increases as the number of titles held increases in a particular subcollection, as a result of satisfying and creating additional demands
3. the expected number of exposures decreases as the set of titles comprising a subcollection ages.

## The Model

Estimates of the parameters, $\alpha$, $\theta$, $\gamma$, $\delta$, and $\eta$ can be determined by performing a nonlinear regression analysis using the model

$$E_{it} = \alpha e^{-\gamma t} N_{it}^{\theta}(X_{1i} + \delta X_{2i})^{\eta} \nu_{it} \qquad (4.47)$$

where $\nu_{it}$ represents a disturbance term. In this analysis a subcollection of a subject area is designated as the unit of association.

Cross-sectional data relating to subcollections of each subject area are utilized descriptive of the number of demands for titles, $E_{it}$, the number of titles held, $N_{it}$, the age of the subcollection, t, and the number of university community members with interests related to the subject area, $X_{1i}$ and $X_{2i}$. Since circulation demand data is utilized to estimate values for the relevant parameters, the resulting value for the scale parameter, $\alpha$, must be augmented by a factor $W_i$, in order to obtain an expression for the expected number of exposures. $W_i$ was defined in the preceding section to be a measure of the equivalent number of exposures per one circulation use. Because demands and exposures are so closely related, the form of the model need not be altered. Further, since we would expect that the demand rate for an "additional title" is relatively low near the optimal value of $N_{it}$, the demand rate and circulation rate for the title are essentially equal. This occurs because virtually all demands are satisfied for titles which experience a relatively low demand rate. The value for $W_i$ is supplied by the decision maker.

Since this model is to be used to estimate the number of exposures experienced by a subcollection of titles over time, it is most desirable to obtain estimates of the parameters utilizing time series data. However, since time series data descriptive of the activities relating to a particular subcollection over time are unavailable, cross-sectional data descriptive of the activities relating to a number of subcollections at a particular point in time are utilized. When using a model for which parameters are estimated from cross-sectional data to predict occurrences over time, it is advisable to identify and explore possible sources of bias resulting from the employment of this technique.

Given estimates for these parameters, the model can be solved to determine the optimal number of new titles to purchase in each subject area. If

$N_{i1}^{o}$ = the optimal number of new titles to purchase (t = 1), in subject area i,

then the value of $N_{i1}^{o}$ is selected such that:

> $\Delta E/\Delta C$ for the purchase of the $N_{i1}^{o\,th}$ title is greater than the exposure-to-cost ratio resulting from the expenditure of marginal funds in other programs; and
>
> $\Delta E/\Delta C$ for the purchase of the $N_{i1}^{o} + 1^{st}$ title is less than the exposure-to-cost ratio resulting from the expenditure of marginal funds in other programs.

We now proceed to determine the optimal value for $N_{i1}^{o}$.

For simplicity, let us assume that the values for $X_{1i}$ and $X_{2i}$ remain constant over the immmediate future years. Then the expected number of exposures experienced by these $N_{i1}$ new titles over the future years is seen to be:

$$\sum_t E_{it} = W_i \alpha N_{i1}^{\theta}(X_{1i} + \delta X_{2i})^{\eta} \sum_t e^{-\gamma t} \qquad (4.48)$$

We sum this expression to infinity to obtain a more tractable solution, although the useful life of a title is probably in the range of 20-30 years. The amount of bias introduced in obtaining this approximate solution can be ascertained when the value of $\gamma$ is estimated.

$$\sum_t E_{it} = W_i \alpha N_{i1}^{\theta}(X_{1i} + \delta X_{2i})^{\eta}\left(\frac{1}{e^{\gamma}-1}\right) \qquad (4.49)$$

The first partial derivative of the total number of exposures with respect to the number of new titles purchased yields:

$$\frac{\partial \sum_t E_{it}}{\partial N_{i1}} = \theta W_i \alpha N_{i1}^{\theta-1}(X_{1i} + \delta X_{2i})^{\eta}\left(\frac{1}{e^{\gamma}-1}\right) \qquad (4.50)$$

This provides a measure of the expected number of additional exposures experienced by the addition of one new title. If we assume that the cost of adding an additional new title is $F_2$, the marginal benefit-cost ratio for adding an additional title becomes

$$\frac{\Delta E}{\Delta C} = \frac{1}{F_2}\left(\frac{\theta \alpha W_i}{e^{\gamma}-1}\right) N_{i1}^{\theta-1}(X_{1i} + \delta X_{2i})^{\eta} \qquad (4.51)$$

If we let

U = the marginal benefit-cost ratio yielded by the expenditure of funds in other programs,

then the following equation can be solved to obtain the optimal number of new titles to purchase in subject area i, $N^{o}_{i1}$.

$$U = \frac{\theta\alpha W_i}{F_2(e^{\gamma}-1)} N_{i1}^{\theta-1}(X_{1i} + \delta X_{2i})^{\eta} \qquad (4.52)$$

Then

$$N^{o}_{i1} = \left(\frac{(\theta\alpha W_i)(X_{1i} + \delta X_{2i})^{\eta}}{(F_2)(e^{\gamma}-1)(U)}\right)^{\frac{1}{1-\theta}} \qquad (4.53)$$

Since we expect that the value of θ lies between zero and one, the exponent of this expression is a positive number greater than one. For changes in the values of the variables in the right hand side of equation 4.53, the value of $N_{it}$ behaves as we might expect.

1. an increase in the number of university community members ($X_{1i}$ and/or $X_{2i}$) specifies an increase in the optimal number of new titles, $Ni_1$
2. an increase in the cost per additional new title, $F_2$, leads to a decrease in $N^{o}_{i1}$

3. an increase in the marginal benefit-cost ratio yielded by other programs, U, leads to a decrease in $N^{o}_{11}$
4. an increase in the value of the parameter which describes the obsolescence effect of knowledge, $\gamma$, leads to a decrease in $N^{o}_{11}$.

If it is deemed desirable to discount the future expected exposures experienced by an additional new title to the current year, the expression for the optimal number of new titles to purchase becomes

$$N^{o}_{11} = \left( \frac{(\theta \alpha W_i)(X_{1i} + \delta X_{2i})^{\tau}}{(F_2)\left(\frac{e^{\gamma}-1}{1+r}\right)(U)} \right)^{\frac{1}{1-\theta}} \qquad (4.54)$$

where r = the value of the exposure discount rate as discussed in Chapter II

Alternatively, the value for the optimal number of new titles to purchase in a given subject area can be expressed as a multiple of the optimal number of new titles purchased in another subject area. This is accomplished by equating the marginal exposure-to-cost ratios, as computed in equation (4.51), for each subject area. For example, to determine the optimal number of new titles to purchase in subject area i as a multiple of the new titles purchased in subject area j, we equate the respective marginal exposure-to-cost ratios.

$$\frac{1}{F_2}\left(\frac{\theta\alpha W_i}{e^{\gamma}-1}\right) N_{it}^{\theta-1}(X_{1i} + \delta X_{2i})^{\eta} = \frac{1}{F_2}\left(\frac{\theta\alpha W_i}{e^{\gamma}-1}\right) N_{jt}^{\theta-1}(X_{1j} + \delta X_{2j})^{\eta} \tag{4.55}$$

Then

$$N_{i1}^{o} = N_{j1}^{o}\left(\frac{X_{1j} + \delta X_{2j}}{X_{1i} + \delta X_{2i}}\right)^{\frac{\eta}{\theta-1}} \tag{4.56}$$

For this example, the results of the model indicate that there should be $\left(X_{1i} + \delta X_{2i}/X_{1j} + \delta X_{2j}\right)^{\eta/\theta-1}$ times as many new titles purchased in subject area i as are purchased in subject area j. In a similar manner this analysis can be extended to determine the relative number of new titles to purchase in all subject areas. Thus, given a specification of the optimal number of new titles to purchase in one subject area, the optimal numbers of new titles to purchase in all other subject areas are determined as a multiple of this number. An estimate of U is still required in this analysis for specifying the optimal number of titles to purchase in the one subject area.

## Conclusion

This concludes the development of a model which provides guidance to administrators in determining the number of new titles to purchase each year for each subject area. Data required to estimate the parameters of the model include the number of titles in a subcollection, the number of demands experienced by a subcollection and the number of university community members with interests relating to a subject area of the subcollection. The model developed here represents one conceptualization relating demand for the use of titles to the purchase of new titles. It might be desirable in future research to explore other variables and relationships in a search for models which might better describe the determinants of demand for the use of titles.

A word of caution is sounded for interpreting the output of this model. The value of the optimal number of new titles to purchase in a given subject area as specified by this model represents a mean or expected value. In determining the actual number of new titles to purchase, the decision maker should not interpret the output of this model in a rote, mechanistic fashion. The decision maker should feel free to purchase a few more or less new titles in response to the particular situation.

## Model For Providing Reserve Documents

Items in a reserve collection undoubtedly are the most used documents in a university library except, perhaps, for current issues of popular general periodicals. Documents are placed on reserve at the request of professors who are teaching courses. Students enrolled in these courses are asked to consult these documents during the course of the semester. The reserve collection is normally confined to a specific room either in which the documents are kept on an open shelf basis or in which the documents are kept on closed shelves requiring a special checking out service for users to gain access.

From a management science viewpoint, the titles provided on reserve and the users required to consult these documents represent a closed system. The titles which are placed on reserve are determined by professors and the number of potential users is determined by the number of students taking courses requiring reserve document reading assignments. For this system the decision variables which influence document exposures are the number of copies of a title provided and the loan period. In this analysis a model is developed to predict the effect upon document exposures of different strategies for the number of copies of a title provided. This model can also be used to predict the effect upon document exposures of different loan periods.

## The Model

In general, the greatest demand for a reserve title occurs during the semester in which the title is actively being used in connection with a course. During this time students are busily involved in writing term papers and preparing for quizzes and examinations in conjunction with the conduct of the course. We focus primarily on the activity during this period of greatest demand to develop a model for determining the number of multiple copies to provide in order to yield a particular level of exposures per unit cost. This can be done with little loss of accuracy since the demand for documents during this period is very great relative to the demand during the remainder of the year. We then make the assumption (which can later be verified) that the selection of the optimum number of copies for each title will be such that virtually no demands for reserve documents will be lost except during the semester in which the title is actively on reserve. We can then focus on the number of expected exposure losses during this period for different multiple copy reserve policies. A determination of $\Delta E/\Delta C$ for different multiple copy strategies will assist in determining the optimal number of copies to provide consistent with the effective allocation of funds on a library-wide basis.

During the semester the demand rate for titles on reserve normally does not vary nearly as much as the demand rate for titles which are allowed to circulate. Except for being slightly below average at the start of the semester and slightly above average at the tail end of the semester, the demand rate for titles on reserve is generally constant during the semester. It does not appear that this slight seasonal variation will significantly affect the determination of the optimal number of copies of a title to place on reserve.

For this analysis data can be collected on the demand rate and the length of time a document is consulted during a period of the semester for a sampling of documents. A regression analysis can then be performed relating demand rate to the number of students enrolled in courses in which the specified document has been designated for reserve. We would expect demand rate for each title to be an increasing monotonic function (possibly linear as depicted in Figure 4.10) with respect to the number of students assigned to read the title. The exact form of this function is determined empirically in Chapter V.

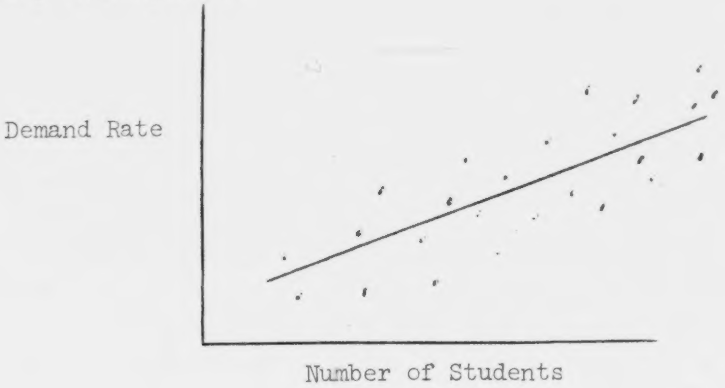

Fig. 4.10--Demand Rate for Titles on Reserve as a Function of the Number of Students in Courses Using These Titles.

Given enrollment figures for a course using a particular book, the library staff can estimate the demand rate for that book during the semester that it is being used in connection with a course. Then, given an estimate of the demand rate for a specific document, the library staff can determine the required number of copies to place on reserve in accordance with a formula which will be developed. If sufficient copies are not available in the library system a decision can be made concerning the purchase of additional copies.

During this period we can assume students demanding a particular document are willing to wait for a period of time if a copy is not readily available. Few students are unwilling to wait at all if they suspect a potential quiz question will draw on material covered by the particular document. Hence, a queuing model will be used which assumes no balking, i.e., a student desiring a document is willing to wait at least for a short period of time. Therefore, given the demand rate during this period for a particular document, we can predict the mean waiting time to use the document when 1, 2, 3, etc. copies are provided. The model describing this situation is outlined below:

$\lambda$ = mean demand rate for use of a particular reserve document

$\mu$ = mean rate at which reserve documents are returned after use ($^1/_\mu$ is the mean time a document is in use by one person)

$P_n(t)$ = probability that n copies of a specific document are in use when $n \leq c$, or that c copies are in use and n-c persons are waiting when $n > c$ at time t

c = number of copies of document available

$$P_0'(t) = -\lambda P_0(t) + \mu P_1(t) \qquad n = 0$$

$$P_n'(t) = -(\lambda + n\mu) P_n(t) + \lambda P_{n-1}(t) + (n+1)\mu P_{n+1}(t) \qquad 1 \le n \le c-1$$

$$P_n'(t) = -(\lambda + c\mu) P_n(t) + \lambda P_{n-1}(t) + c\mu P_{n+1}(t) \qquad n \ge c \qquad (4.57)$$

Assuming steady state conditions, we obtain:

$$P_o = \frac{1}{\sum_{r=0}^{c-1} \frac{(c\rho)^r}{r!} + \frac{(c\rho)^c}{c!}\left(\frac{1}{1-\rho}\right)} \qquad n = 0$$

$$P_n = \frac{1}{n!}(c\rho)^n P_o \qquad 1 \le n \le c-1$$

$$P_n = \frac{c^c}{c!}(\rho)^n P_o \qquad n \ge c \qquad (4.58)$$

where

$$\rho = \frac{\lambda}{\mu c} \qquad (4.59)$$

The probability that a user will have to wait greater than $\tau$ time units when the mean demand rate for a document is $\lambda_o$, mean return rate is $\mu$, and $c_o$ copies are provided is:

$$P(t > \tau | \lambda = \lambda_o, c = c_o, \mu = \mu_o) = W(t) \qquad (4.60)$$

We now proceed to obtain this complement of a probability distribution function.

The probability that a user arrives to find all c copies in use and r users waiting ahead of him to use the document is

$$P_{c+r} = \frac{c^c}{c!} \rho^{(c+r)} P_o \qquad (4.61)$$

The rate at which users return their copies when finished is Poisson distributed given that individual usage times are exponential and all copies are in use. Therefore, the probability that j users return j copies prior to time $\tau$ is

$$P\left(\begin{matrix}\text{j users return j copies} \\ \text{prior to time } \tau\end{matrix}\right) = \frac{e^{-c\mu\tau} (c\mu\tau)^j}{j!} \qquad (4.62)$$

Therefore, if a user arrives to find r users in the queue ahead of him, the probability that he waits longer than time $\tau$ to use a copy is

$$P(t > \tau | r + c) = \sum_{j=0}^{r} \frac{e^{-c\mu\tau} (c\mu\tau)^j}{j!} \qquad (4.63)$$

The unconditional probability that a user must wait longer than time $\tau$

to use a copy of the document is

$$P(t > \tau) = \sum_{r=0}^{\infty} P_{r+c} P(t > \tau | r+c) \tag{4.64}$$

$$= \sum_{r=0}^{\infty} \frac{c^c}{c!} \rho^{r+c} P_o \sum_{j=0}^{r} \frac{e^{-c\mu\tau}(c\mu\tau)^j}{j!}$$

$$= \frac{(\rho c)^c}{c!} P_o\, e^{-c\mu\tau} \sum_{r=0}^{\infty} \rho^r \sum_{j=0}^{r} \frac{(c\mu\tau)^j}{j!}$$

$$= \frac{(\rho c)^c}{c!} P_o\, e^{-c\mu\tau}\Big(1 + \rho(1 + c\mu\tau) + \rho^2\Big(1 + c\mu\tau + \frac{(c\mu\tau)^2}{2}\Big) + \ldots\Big)$$

$$= \frac{(\rho c)^c}{c!} P_o\, e^{-c\mu\tau} \frac{e^{\rho\mu c\tau}}{1-\rho}$$

$$P(t > \tau) = \frac{(\rho c)^c P_o}{(1-\rho)c!} e^{-(c\mu(1-\rho)\tau)} \tag{4.65}$$

We note that this is the complement of a cumulative probability distribution function. The density function for the time spent in the queue is as follows:

$$W(t) = \delta(t) \sum_{n=0}^{c-1} \frac{(c\rho)^n}{n!} P_o + \frac{c\mu(\rho c)^c}{c!} P_o\, e^{-c\mu(1-\rho)t} \tag{4.66}$$

The first term represents the probability that the waiting time is zero whereas the second term represents the waiting time probability density for $t > 0$.

In general, we would expect that the average user is willing to wait not more than a certain length of time to use a copy of a particular reserve title. If we assume that the average user is willing to wait no more than a period of time, $\tau$, we can determine the $P(t > \tau)$ given various values of c. For a particular value of $c = c_o$, the expected number of lost exposures is then $P(t > \tau | c_o)$ times $\lambda$. For $c = c_o + 1$, the expected number of lost exposures is then $P(t > \tau | c_o + 1)$ times $\lambda$. Hence, in this range, the expected number of exposures gained when one more copy of the $k^{th}$ title is provided is shown by

$$\Delta E_k = m\lambda_k \left[P(t > \tau | c_o) - P(t > \tau | c_o + 1)\right], \qquad (4.67)$$

where m = the number of years the title k is expected to be on reserve and $\lambda_k$ is the demand rate for title k during the semester. A measure of the incremental cost of purchasing, processing, controlling, etc. one additional copy of title k is determined to be $F_3$. Therefore, the benefit-cost ratios, $\Delta E_k / \Delta C_k$, can be computed for various reserve duplicate copy policies. A decision rule can be formulated to determine the optimum number of copies to provide for each title. The rule would be of the form:

Provide c* copies of a reserve title such that:

$$\frac{m\lambda_k \left[P(t > \tau | c^* - 1) - P(t > \tau | c^*)\right]}{F_3} \geq U$$

$$\frac{m\lambda_k \left[P(t > \tau | c^*) - P(t > \tau | c^* + 1)\right]}{F_3} < U \qquad (4.68)$$

where U is a value of the marginal benefit-cost ratio yielded by the expenditure of funds in other areas.

Prior to the employment of this model it is important to ascertain its sensitivity as a function of certain assumptions and approximations made in the course of this development.

What remains is the determination of the values of $1/\mu$ and $\lambda$. An estimate of $\mu$ can be determined as a function of the mean use period for a sample of uses. While the determination of the mean use rate is rather straightforward, the determination of $\lambda$ is not quite so direct. In the case in which a check out system is utilized, $\lambda$ for each document is determined by counting the number of check outs for all copies of that document in the sample. For the case in which no check out procedure is utilized, determination of $\lambda$ for each sampled document is slightly more complex. In this case the reserve room can be surveyed at random times during the semester to determine the number of copies of each sampled document not present on the shelf at that time. We let

f = average number of copies off the shelf during times sampled.

$\mu$ is expressed in terms of the number of use periods in the period and $\lambda$ is the demand rate. Then the mean demand rate for a title can be estimated as the product of the average number of copies off the shelf and the number of use periods during the period in question.

$$\lambda = (f)(\mu) \tag{4.69}$$

Knowing the values of f, and $^1/\mu$, we can estimate $\lambda$ for each document sampled. A regression analysis can relate values of $\lambda$ obtained with numbers of students enrolled in courses using the documents.

Fortunately the equation for $P(t > \tau)$ need not be solved individually for each different demand rate $\lambda$, to determine the number of copies required to meet the desired criteria. Molina[20] has provided charts to determine the solution to this problem for $c = 1, 2, 3, 4, 5, 7, 10, 15, 20$. From the charts, knowing the values of $\tau/\mu$, $\lambda\mu/c$, and the desired probability $P(t > \tau)$, we can select the value of c which meets the criterion. This eliminates much tedious computation and simplifies application of this model.

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[20] Edward Molina, "Application of the Theory of Probability to Trunking Problems," The Bell System Technical Journal, 6:461-494 (1927)

## Cautions and Conclusions

In the development of the model it was assumed that no balking occurred until the waiting time to use the document exceeded a certain time, for example, one hour. This assumption may not be completely tenable since a student desiring a document may decide that the wait is too long or on the other hand, he may be willing to wait much longer. Some students may decide the wait is too long and decide to come back at another time. If this occurs, arrivals are not strictly Poisson distributed. The sensitivity of the model to these occurrences can be tested empirically.

In the context of this model, the mean demand rate for any particular title is a random variable. Sometimes it is greater than the predicted mean demand rate which is derived from the regression analysis for use in this model, sometimes it is less. Therefore, the preceding probability criterion is an approximation. The accuracy of the approximation is dependent upon the magnitude of the variance of the individual title mean demand rates about the regression line.

In the regression model used for predicting the expected demand rate for document use as a function of the number of students assigned to read the document, actual reserve use data were employed as opposed to actual reserve demand data. Hence, we would expect that a prediction of mean demand rate yielded by this model would be less than the actual mean demand rate, by a factor descriptive of the actual rate of lost demands. If the rate of lost demands is small when data for this model are collected, the differences between actual demand rates and predicted demand rates will be negligible.

This concludes the development of a model to provide guidance to the library administrator in determining an effective strategy for the placing of multiple copies of a title on reserve.

## Summary

In this chapter analytical models were developed descriptive of user-library interaction related to the program of Providing Access to Documents Within the Library. Initially a model was developed for estimating the demand rate for circulation use of titles as a function of actual circulation rate and variables descriptive of research and teaching programs of the university. Then a model was developed which relates the demand rate for circulation use of titles with the demand rate for in-library use of titles. Two separate models were developed for determining the optimal strategy for the purchase of new titles and the optimal strategy for the purchase of multiple copies for these titles. Finally a model was developed for determining an effective strategy for placing multiple copies of titles on reserve which have been designated as required reading material by the professor of the course.

The development of the preceding analytical models represents a sampling of the type of models which can be developed and utilized in the planning and decision-making process. In each case the output of the model yielded results which could be used in the construction of benefit-cost ratios. The values of these ratios can then be utilized in the decision-making and planning process to assist in the effective allocation of funds.

In the next chapter the preceding models are validated and parameters are computed based upon data collected relating to the Lippincott Library. These data are descriptive of the university population utilizing the library, holdings of the library, and the use made of the library during a period in the Spring Semester of 1970.

# CHAPTER V

# MODEL APPLICATIONS

## Introduction

In this chapter the applicability and validity of the decision models developed in the preceding chapter are verified. For this phase of the research data were gathered relevant to the activities of the Lippincott Library at the University of Pennsylvania. The primary mission of Lippincott Library is to serve the needs of students and faculty associated with the Wharton School of Finance and Commerce. Lippincott Library has a primary responsibility for meeting user needs in the subject categories of

1. Accounting
2. Computer and Information Science
3. Economics
4. Finance
5. Industrial Management
6. Industrial Relations
7. Insurance
8. International Business
9. Business Law
10. Marketing
11. Statistics and Operations Research
12. Transportation and Public Utilities

Van Pelt Library maintains a sizable collection in Economic Theory and Economic History in addition to that held by Lippincott Library in these areas. Responsibility for Statistics, Operations Research, and Computer Science are shared by other departmental libraries on campus. In addition, a sizable collection of Insurance books is held by the Heubner Foundation which is not part of the library, yet which is readily accessible to Insurance majors and faculty members.

Data descriptive of user behavior, library resources, and university membership with interests in these areas were collected during a period in the Spring Semester of 1970. A detailed description of the data and methods of collection is included in Appendices C and H.

In this chapter these data are used to test, modify and estimate parameters for the models developed ~~in the~~ preceding chapter to assist decision making and planning in the major areas of:

1. Forecasting demand
2. Multiple copy selection
3. New title selection
4. Reserve service

An analysis is performed on each of these major areas.

## Forecasting Demand

In this section the components comprising the Circulation Demand Model are analyzed utilizing the data collected relevant to the activities of this area. The components analyzed or estimated for the application of this model include:

1. estimation of mean loan period ($^1/\mu$)
2. verification that the demand distribution, $P(\lambda)$ is a truncated exponential distribution with mean demand rate, $\overline{D}$
3. determination of the form and parameters for the conditional probability distribution for the number of multiple copies given demand rate, $P(c|\lambda)$
4. solution of the model for the mean demand rate $\overline{D}$, as a function of circulation rate, $\overline{G}$
5. regression of circulation uses and in-library uses to determine the factor, $\beta$, relating these two variables
6. multiple regression of $\overline{D}$ as a function of university membership.

Each of these topics is now considered in succession.

To obtain an estimate of $^1/\mu$, the mean loan period, a one-week sample was collected of the number of circulations, G, and the number of documents out of the library, L, at this time. The data obtained for this week were as follows:

| | |
|---|---|
| Number of circulations (G) | 223 |
| Number of books out (L) | 655 |

We assume that the mean loan period is similar among subject areas, which permits an aggregation of data to yield a single estimate of $^1/\mu$. Using the results of the model developed in Chapter IV (equation 4.12), the best estimate of $^1/\mu$ is $^L/G = {}^{655}/223 = 2.94$ weeks. This is somewhat longer than the stated two-week loan period currently employed by the Lippincott Library. However, when we consider that the two-week period is enforced for students but not for faculty, we would expect that the actual mean loan period might exceed the stated loan period.

A sampling of titles in each subject area was performed to obtain an estimate of the circulation rate per title over the previous eight year period. These data were fit to a truncated exponential distribution of the type depicted in equation 4.5. An assumption is made that the form of the demand function is similar to the circulation function which allows us to verify the form of the demand function utilizing circulation data. A Kolmogorov-Smirnov goodness of fit test was conducted on these data for each subject category. The results of this test at a 5% level of significance support the hypothesis that these data can be described by a truncated exponential distribution. The results of these tests are presented as Table 5.1.

TABLE 5.1

KOLMOGOROV-SMIRNOV GOODNESS OF FIT TEST OF CIRCULATION DATA FIT TO A TRUNCATED EXPONENTIAL PROBABILITY DISTRIBUTION FOR EACH SUBJECT CATEGORY

| SUBJECT CATEGORY | $P_o$ | $\overline{R}$ | n | MAXIMUM ABSOLUTE DIFFERENCE | K-S CRITICAL VALUE (.05) | DISTRIBUTION VERIFIED |
|---|---|---|---|---|---|---|
| Marketing | .24 | 1.45 | 38 | .076 | .218 | Yes |
| Industrial Mgmt. | .19 | 1.36 | 32 | .105 | .238 | Yes |
| Insurance | .56 | .42 | 25 | .083 | .264 | Yes |
| Accounting | .32 | 1.67 | 25 | .065 | .264 | Yes |
| International Bus. | .44 | .76 | 25 | .103 | .264 | Yes |
| Industrial Relat. | .36 | .40 | 25 | .060 | .264 | Yes |
| Economics | .48 | .52 | 25 | .082 | .264 | Yes |
| Statistics-O.R. | .35 | 3.14 | 26 | .100 | .260 | Yes |
| Finance | .44 | 1.23 | 25 | .056 | .264 | Yes |
| Trans. & Pub. Util. | .36 | .81 | 25 | .057 | .264 | Yes |
| Business Law | .52 | .59 | 29 | .049 | .246 | Yes |
| Computer Science | .52 | 1.13 | 25 | .064 | .264 | Yes |

Note: $P_o$ is estimated by the proportion of titles in the sample of size n which have experienced no circulations in an eight year period. $\overline{R}$ is estimated by the mean circulation rate of the titles sampled which have circulated in the past eight year period.

The next step in the application of this Circulation Demand Model involves the fitting of data relating recorded circulations and the number of copies of a title to a theoretical probability distribution of the type depicted in Figure 4.1 Values of the yearly circulation rate for each title, R, and the number of copies, c, were collected for a random sample of 320 titles. A $\chi^2$ test for independence of the data, classified by intervals of R and values of c, revealed no significant relationship between R and c, as shown in Table 5.2.

Since we assume there is a high correlation between the demand rate for a title, $\lambda$, and the mean circulation rate for a title, R, we would further conclude that there is no significant relationship between $\lambda$ and c. This result is somewhat discouraging as it indicates that for this library the purchase of multiple copies for a title is independent of the demand rate experienced by that title. At present Lippincott Library has no policy for ordering additional copies of a book if a title is out and additional reserves are placed for it. If such a policy were in effect, a relationship between $\lambda$ and c would undoubtedly emerge ex post facto. Another reason for this lack of dependence is that a great number of the more popular titles are used in conjunction with a course.

# TABLE 5.2

## $\chi^2$ TEST FOR INDEPENDENCE BETWEEN THE MEAN CIRCULATION RATE, R, AND THE NUMBER OF COPIES, c, FOR A SAMPLE OF 320 TITLES

| **Mean Yearly** Circulation Rate Per Title (R) | Number of Copies (c) | | Total |
|---|---|---|---|
| | 1 | 2 or More | |
| 0 | 112<br>(109.8) | 13<br>(15.2) | 125 |
| .10- .50 | 64<br>(65.0) | 10<br>(9.0) | 74 |
| .51-1.00 | 35<br>(37.8) | 8<br>(5.2) | 43 |
| 1.01-2.00 | 35<br>(33.4) | 3<br>(4.6) | 38 |
| 2.01 or more | 35<br>(35.1) | 5<br>(4.9) | 40 |
| Total | 281 | 39 | 320 |

$\chi^2 = \sum \frac{(O-E)^2}{E} = 2.830$ $\chi^2_{.05,\ 4} = 9.49$

NOTE: In this table the numbers in parentheses under the observed frequencies represent the expected frequencies of occurrence if R and c were independent.

These titles are placed on reserve and are not included in the set of titles allowed to circulate. The set of titles included in the analysis in Table 5.2 consists mainly of titles which experience a low to moderate rate of demand. We would expect that it would be a difficult task to determine which of these titles are expected to experience the greatest demand rates and hence, are the best candidates for which to purchase a multiple copy. In view of these results it is assumed that $P(c|\lambda) = P(c)$ for the remainder of this analysis. Hence, the values in the bottom row of Table 5.2 are used to compute the probabilities taken on by the distribution, $P(c)$. In this instance $P(c=1) = 281/320 = .878$ and $P(c=2) = 39/320 = .122$. Since only two titles had 3 or more copies, we assume $P(c \geq 3) \doteq 0$ for expediency in this analysis.

Since the form of the distribution for $P(\lambda)$ has been verified as exponential, values for the distribution of multiple copies, $P(c)$, have been determined, and a value for $\mu$ computed, the Circulation Demand Model developed in Chapter IV can be solved to yield $\overline{D}_i$, the mean demand rate for titles in subject are i. Obtaining an exact solution for $\bar{D}_i$, the mean demand rate, in the Circulation Demand Model of equation (4.13) proves to be rather difficult. An approximate solution was obtained by fitting a polynomial function to each term of the model. Values for each term were obtained as a function of $\bar{D}_i$ by numerical integration. Polynomial functions were then fit to these values yielded by the numerical integration. The goodness of these approximations can be found in Appendix D. Within the relevant range of $\bar{D}_i$ for titles in the Lippincott collection, the following functions provide good approximations to the numerically integrated values for $1/\mu = 2.94$.

$$\frac{1}{\bar{D}} \int_0^\infty \left( \frac{\lambda^2/\mu}{1 + \lambda/\mu} \right) e^{-\lambda/\bar{D}} \, d\lambda \doteq \frac{2\bar{D}^2}{\mu} - \frac{\bar{D}^3}{3\mu} + \frac{3}{5}\frac{\bar{D}^4}{\mu^2} - \frac{3\bar{D}^5}{10\mu^2}$$

$$(5.1)$$

$$\frac{1}{\bar{D}} \int_0^\infty \left( \frac{\lambda^3/2\mu^2}{1 + \lambda/\mu + \lambda^2/2\mu} \right) e^{-\lambda/\bar{D}} \, d\lambda \doteq \frac{3\bar{D}^3}{\mu^2} - \frac{2}{3}\frac{\bar{D}^4}{\mu^2} + \frac{11}{10}\frac{\bar{D}^5}{\mu^3} - \frac{2\bar{D}^6}{5\mu^4}$$

Since $P(c \geq 3) \doteq 0$ in this analysis, polynomial functions were not fit to the terms corresponding to 3 copies, 4 copies, etc. The expression for the circulation model then becomes

$$G_i = N_i(1-p_o)\bar{D}_i - N_i(1-p_o)\left\{P(c=1)\left[\frac{2\bar{D}^2}{\mu} - \frac{\bar{D}^3}{3\mu} + \frac{3}{5}\frac{\bar{D}^4}{\mu^2} - \frac{3\bar{D}^5}{10\mu^3}\right]\right.$$

$$\left. + P(c=2)\left[\frac{3\bar{D}^3}{\mu^2} - \frac{2}{3}\frac{\bar{D}^4}{\mu^2} + \frac{11}{10}\frac{\bar{D}^5}{\mu^3} - \frac{2\bar{D}^6}{5\mu^4}\right]\right\} \qquad (5.2)$$

Estimates of the number of titles held in each subject area, $N_i$, were obtained by sampling title shelf list cards within the relevant spans of L. C. and Modified D. D. numbers. To obtain these estimates on a per subject area basis, we found it necessary to determine the appropriate Library of Congress and Lippincott Library Modified Dewey Decimal Classification Numbers corresponding to each subject area. The results of this effort to assign call numbers to relevant subject areas for Lippincott Library are presented in Appendix E. Estimates of the number of titles held in each subject area are listed in Table 5.4. The method of sampling and the estimation technique are included in Appendix C.

Weekly seasonal indices were computed of circulation activity for Lippincott Library. These indices were computed based upon weekly circulation data over a three year period. These results are presented in Table 5.3.

TABLE 5.3

WEEKLY SEASONAL INDICES OF CIRCULATION ACTIVITY FOR LIPPINCOTT LIBRARY WHERE WEEK 1 REPRESENTS THE FIRST WEEK OF THE FALL SEMESTER

| WEEK | REMARKS | TOTAL WEEKLY CIRCULATIONS 1968 | 1969 | 1970 | SEASONAL INDEX S.I. |
|---|---|---|---|---|---|
| 1 | Begin Fall Semester | 110 | 184 | 206 | .6812 |
| 2 | | 207 | 166 | 221 | .8112 |
| 3 | | 229 | 208 | 295 | 1.0036 |
| 4 | | 249 | 297 | 292 | 1.1492 |
| 5 | | 228 | 251 | 278 | 1.0348 |
| 6 | | 252 | 224 | 337 | 1.1128 |
| 7 | | 295 | 227 | 341 | 1.1804 |
| 8 | | 300 | 276 | 391 | 1.3260 |
| 9 | | 409 | 236 | 380 | 1.4040 |
| 10 | | 463 | 330 | 366 | 1.5600 |
| 11 | | 473 | 265 | 460 | 1.6692 |
| 12 | | 690 | 270 | 345 | 1.7888 |
| 13 | | 713 | 279 | 406 | 1.9136 |
| 14 | End Fall Semester | 442 | 261 | 243 | 1.3000 |
| 15 | | 303 | 162 | 163 | .5840 |
| 16 | | 180 | 69 | 72 | .4368 |
| 17 | | 145 | 104 | 79 | .4472 |
| 18 | | 75 | 110 | 159 | .4680 |
| 19 | Begin Spring Semester | 119 | 212 | 253 | .6008 |
| 20 | | 144 | 340 | 289 | 1.0556 |
| 21 | | 306 | 263 | 306 | 1.2272 |
| 22 | | 350 | 264 | 361 | 1.3884 |
| 23 | | 393 | 266 | 390 | 1.4352 |
| 24 | | 415 | 243 | 362 | 1.3936 |
| 25 | | 435 | 260 | 457 | 1.5912 |
| 26 | | 379 | 379 | 524 | 1.7576 |
| 27 | Spring Recess | 193 | 265 | 250 | .9568 |
| 28 | | 372 | 309 | 481 | 1.6068 |
| 29 | | 403 | 332 | 400 | 1.5548 |
| 30 | | 304 | 382 | 407 | 1.4976 |
| 31 | | 354 | 376 | 438 | 1.6016 |
| 32 | | 345 | 311 | 427 | 1.4820 |
| 33 | | 369 | 357 | 436 | 1.5912 |
| 34 | End Spring Semester | 309 | 255 | 314 | 1.2012 |
| 35 | | 208 | 177 | 174 | .7644 |
| 36 | | 124 | 117 | 123 | .4992 |
| 37 | | 119 | 150 | 124 | .5356 |
| 38 | | 120 | 134 | 137 | .5356 |
| 39 | | 156 | 170 | 221 | .7488 |
| 40 | | 100 | 124 | 155 | .5148 |
| 41 | | 109 | 157 | 212 | .6552 |
| 42 | | 85 | 121 | 123 | .4472 |
| 43 | | 74 | 158 | 84 | .4316 |
| 44 | | 97 | 192 | 142 | .5576 |
| 45 | | 135 | 162 | 220 | .7072 |
| 46 | | 121 | 94 | 142 | .4608 |
| 47 | | 118 | 122 | 153 | .5356 |
| 48 | | 122 | 143 | 114 | .5148 |
| 49 | | 82 | 69 | 120 | .3072 |
| 50 | | 76 | 117 | 128 | .4368 |
| 51 | | 51 | 93 | 56 | .2120 |
| 52 | | 83 | 136 | 99 | .4316 |
| | | 13,003 | 11 189 | 13,708 | |

Note: Data listed in this table were obtained from records maintained by Lippincott Library.

During a two-week period in the Spring Semester, 1970, the numbers of circulations in each subject area were recorded. An estimate of the average number of weekly circulations was obtained by dividing the sum of the number of circulations in each subject area obtained in the two-week sampling period, by the sum of the seasonal indices for the corresponding two weeks. Estimates of the numbers of circulations each week of the year for each subject area were obtained by multiplying the average number of weekly circulations by the appropriate value of the seasonal index. Since the Circulation Demand Model was constructed in terms of yearly data, certain manipulations were required to obtain the desired estimate of the mean yearly demand rate for titles in each subject area, $\overline{D}_i$. To perform this analysis, the estimates of the number of circulations each week of the year are multiplied by 52. These values are then used as estimates for $G_i$ in equation (5.2). For each subject area this equation is then solved 52 times yielding 52 estimates of the mean yearly circulation rate. Summation of these 52 estimates and subsequent division by 52, yields an estimate of $\overline{D}_i$, the mean yearly demand rate for titles in subject area i.

Pooled estimates for the following parameters were used in this analysis for each subject area:

$\mu = 52/2.94 = 17.6$ use periods per year

$p_o = .407$ (5.3)

$P\ (c=1) = .878$

$P\ (c=2) = .122$

Pooled estimates were used here as insufficient data were collected to provide reliable estimates for the parameter values on an individual subject area basis.

This procedure of solving the Circulation Demand Model for weekly values and summing over a year may seem excessively tedious. However, if there is considerable seasonal variation in the circulation rate and if good approximations of the yearly demand and unsatisfied demand rates are desired, it is necessary to proceed in this manner. The effort required to utilize this procedure is significantly reduced by the employment of a computer polyroot subroutine. Estimates of the mean yearly demand rate, circulation rate, unsatisfied demand rate and number of unsatisfied demands for

TABLE 5.4

ESTIMATED NUMBER OF TITLES HELD, $N_i$, MEAN YEARLY DEMAND RATE $\overline{D}_i$, MEAN YEARLY CIRCULATION RATE, $\overline{C}_i$, MEAN YEARLY UNSATISFIED DEMAND RATE, $\overline{D}_i$-$\overline{C}_i$, AND THE NUMBER OF UNSATISFIED DEMANDS FOR TITLES IN EACH SUBJECT AREA

| SUBJECT AREA i | $N_i$ | $\overline{D}_i$ | $\overline{C}_i$ | $\overline{D}_i-\overline{C}_i$ | NUMBER OF UNSATISFIED DEMANDS |
|---|---|---|---|---|---|
| Accounting-Financial | 371 | 0.922 | 0.840 | 0.082 | 18.0 |
| Accounting-Managerial | 398 | 0.966 | 0.868 | 0.098 | 23.1 |
| Business Law | 398 | 0.634 | 0.590 | 0.044 | 10.4 |
| Computer Science | 318 | 0.879 | 0.815 | 0.064 | 5.2 |
| Economics-Theory | 399 | 1.261 | 1.143 | 0.118 | 27.9 |
| Econometrics | 120 | 1.567 | 1.381 | 0.186 | 13.2 |
| Economic History | 216 | 0.539 | 0.511 | 0.028 | 3.6 |
| Economics-Systems | 202 | 0.282 | 0.273 | 0.009 | 1.1 |
| Economics-Eco. & Soc. Data | 614 | 0.430 | 0.409 | 0.021 | 7.7 |
| Eco.-Growth, Dev. & Plan. | 4071 | 0.706 | 0.655 | 0.051 | 123.0 |
| Economics-Agricultural | 467 | 0.399 | 0.381 | 0.018 | 5.0 |
| Economics-Industries | 4180 | 0.329 | 0.316 | 0.013 | 32.2 |
| Eco.-Fiscal & Mon. Theory | 347 | 0.435 | 0.416 | 0.019 | 3.9 |
| Eco.-Welfare Program | 195 | 0.316 | 0.299 | 0.017 | 2.0 |
| Eco.-International | 485 | 1.567 | 1.364 | 0.203 | 58.3 |
| Eco.-Fis. Pol. & Pub. Fin. | 1577 | 0.240 | 0.234 | 0.006 | 5.6 |
| Finance-Real Estate | 351 | 0.248 | 0.241 | 0.007 | 1.5 |
| Finance-Money | 339 | 0.411 | 0.395 | 0.016 | 3.2 |
| Finance-Business Fin. | 1575 | 0.770 | 0.713 | 0.057 | 53.2 |
| Finance-Banking | 975 | 0.216 | 0.211 | 0.005 | 2.9 |
| Finance-Trust & Estate | 503 | 0.363 | 0.349 | 0.014 | 4.2 |
| Ind. Relations-Labor Rel. | 2626 | 0.700 | 0.648 | 0.052 | 80.9 |
| Ind. Relations-Personnel Rel. | 314 | 0.916 | 0.829 | 0.087 | 16.2 |
| Ind.-Operations. Manag. | 1363 | 0.757 | 0.704 | 0.053 | 42.8 |
| Ind.-Business Organization | 1367 | 0.970 | 0.882 | 0.088 | 71.3 |
| Insurance-Business | 143 | 0.404 | 0.387 | 0.017 | 1.4 |
| Ins.-Life & Health | 212 | 0.560 | 0.531 | 0.029 | 3.6 |
| Insurance-Social | 793 | 0.075 | 0.074 | 0.001 | .5 |
| Ins.-Property & Liability | 212 | 0.916 | 0.829 | 0.087 | 10.9 |
| Insurance-General | 93 | 0.331 | 0.320 | 0.011 | .6 |
| International Business | 635 | 0.091 | 0.090 | 0.001 | .4 |
| Marketing-Management | 500 | 0.623 | 0.581 | 0.042 | 12.4 |
| Marketing-Research | 73 | 0.836 | 0.784 | 0.052 | 2.2 |
| Marketing-Consumer Behavior | 40 | 0.802 | 0.742 | 0.060 | 1.9 |
| Marketing-Promotion & Distrib. | 701 | 0.704 | 0.650 | 0.054 | 22.4 |
| Marketing-Theory & General | 486 | 0.741 | 0.698 | 0.043 | 12.4 |
| Stat. & O. R.-Statistics | 329 | 3.888 | 2.792 | 1.096 | 213.7 |
| Stat. & O.R. Operations Research | 237 | 3.404 | 2.518 | 0.886 | 124.2 |
| Trans. & Pub. Util.-Traffic Mgmt. | 201 | 0.139 | 0.137 | 0.002 | .2 |
| Trans. & Pub. Util.-Trans. Ind. and Reg. | 1358 | 0.406 | 0.388 | 0.018 | 14.5 |
| Trans. & Pub. Util.-Util. Ind. and Reg. | 618 | 0.192 | 0.188 | 0.004 | 1.5 |
| ALL AREAS | | 0.618 | 0.559 | 0.059 | 1039.2 |

titles in each subject area for the Lippincott Library are depicted in Table 5.4. These estimates of the number of unsatisfied demands include only the frustrated demands which occur as a result of a user requesting to borrow a copy of a title for which all copies are currently out on loan. This does not include a figure for the unsatisfied demands resulting when a user wishes to consult a copy of a title which is not part of the library's collection. Nor does it include a figure for the unsatisfied demands resulting from demands for copies of a title which are lost, stolen, mis-shelved or in transit.

In Chapter IV we noted that the Circulation Demand Model yields weekly estimates of $\overline{D}$, which are overestimates as the circulation rate increases and are underestimates as the circulation rate decreases over time. Another model was developed based on the number of books out during any week which yields underestimates of $\overline{D}$, as the number of documents out increase and overestimates of $\overline{D}$ as the number of books decrease over time. The rationale for developing this second model was to obtain a prediction of the bias in the estimates of $\overline{D}$ yielded by the Circulation Demand Model.

The model developed appears as follows:

$$L = \frac{N}{\overline{D}} (1-p_o) \int_o^{\infty} \left[ P(c=1) \left( \frac{\lambda/\mu}{1 + \lambda/\mu} \right) + P(c=2) \left( \frac{\lambda/\mu + \lambda^2/\mu^2}{1 + \lambda/\mu + \lambda^2/2\mu^2} \right) \right] e^{-\lambda/\overline{D}} d\lambda \qquad (5.4)$$

where L = the number of documents out of the library on loan at a particular point in time.

Since the probability of 3 copies of a title is insignificant, we ignore the term associated with this occurrence.

The determination of an exact solution for this integral such that the expression can be solved for $\overline{D}$ proves to be rather difficult. Approximations were generated via the same technique used to solve the previous model as follows:

For $1/\mu = 2.94$

$$\frac{1}{\bar{D}}\int_0^\infty \frac{\lambda/\mu}{1+\lambda/\mu}\, e^{-\lambda/\bar{D}}\, d\lambda \doteq \frac{\bar{D}}{\mu} - \frac{2\bar{D}^2}{\mu^2} + \frac{3\bar{D}^3}{\mu^3}$$

$$(5.5)$$

$$\frac{1}{\bar{D}}\int_0^\infty \frac{\lambda/\mu + \lambda^2/\mu^2}{1+\lambda/\mu+\lambda^2/2\mu^2}\, e^{-\lambda/\bar{D}}\, d\lambda \doteq \bar{D}/\mu + 3\bar{D}^2/\mu^2$$

Values yielded by these approximations as compared to the simulated values are listed in Appendix D. Therefore, solution of the following expression yields alternative estimates of the mean demand rate per title, $\bar{D}$.

$$L = (1-p_o)N\left[P(c=1)\left(\frac{\bar{D}}{\mu} - \frac{2\bar{D}^2}{\mu^2} + \frac{3\bar{D}^3}{\mu^3}\right) + P(c=2)\left(\frac{\bar{D}}{\mu} + \frac{3\bar{D}^2}{2\mu^2}\right)\right] \qquad (5.6)$$

In this regard to obtain an estimate of the magnitude of the bias, counts were recorded of the number of books out and the number of circulations for all subject areas during a one-week period in the early part of the Spring Semester, 1971. During this period the circulation rate is increasing over time. A total of 223 books circulated during this week and 655 books were out at week's end. Therefore, letting $L = 655$, and $N = \Sigma N_i = 30{,}546$, and using the values of $P(c=1)$, $P(c=2)$ and $p_o$ listed in equation 5.3, solution of the above model yields $\bar{D} = .650$. Further, letting $G = 223$ and using the values for N, P(c=1), P(c=2) and $p_0$ as defined, solution of the Circulation Demand Model yields an estimate of $\overline{D} = .683$. As expected, this estimate is slightly higher than the previous estimate of $\overline{D}$. These two measures bracket the true mean value of demand with .683 being an overestimate and .650 being an underestimate. Utilization of the estimated demand rate generated by the Circulation Demand Model yields a maximum error of $(.683-.650)/.683 \approx 4.9\%$ within this range of demand rates. The impact of this bias becomes less significant when the demand rates are aggregated over the year since the bias is positive when demand is rising and negative when demand is decreasing. In this case the positive biases have the effect of nullifying the negative biases resulting in an even more accurate estimate of $\overline{D}$. From this we conclude that estimates of the yearly demand rate yielded by the model which utilizes circulation data are sufficiently accurate for this analysis.

Up to this point only circulation uses of documents have been considered. These same documents also experience uses in the library which do not result in recorded circulations. A correlation analysis relating $\overline{H}_i$, the mean yearly rate at which titles in subject area i are used within the library, and $\overline{G}_i$, the mean yearly rate at which titles in subject area i are circulated from the library, was performed. A subject area was used as the unit of association with data collected descriptive of the circulation and in-library use rate for each of 39 subject areas. To obtain the data, counts were recorded of the number of circulations and the number of titles used in the library during a two week period. In-library uses were obtained by recording the call numbers of titles left out on tables and desks during a period in which users were instructed not to reshelve documents. Utilization of the following linear model

$$\overline{H}_i = \alpha + \beta\overline{G}_i + \varepsilon_i \qquad \text{where } i = 1, 2, \ldots, 39 \qquad (5.7)$$

yielded the following estimates for the parameters

$$\hat{\alpha} = .0165$$

$$\hat{\beta} = .9732$$

with a coefficient of correlation, $r = .924$. This correlation, which is highly significant ($r_{.05} = .296$), substantiates our belief that a strong relationship exists between the rate of circulation and rate of in-library use of titles for various subject areas. From this analysis it can be concluded that there is approximately one in-library use for each circulation use in the Lippincott Library.

Since $\overline{G}_i$ and $\overline{D}_i$ are highly correlated, it can be assumed that on the average there are $\beta$ demands for in-library use of a title for each demand for circulation use of that title. Therefore, the mean demand rate for in-library use of titles in subject area i can be predicted as a function of the demand rate for circulation use, $\beta\overline{D}_i$, once the value of the parameter $\beta$ is established. The mean demand rate for both in-library and circulation use becomes

$$\overline{D}_i + \beta\overline{D} = \overline{D}_i(1+\beta). \tag{5.8}$$

Finally, the estimated rate of unsatisfied demands for both circulation use and in library use is $(1 + \beta)(\overline{D}-\overline{G})$. The estimated numbers of unsatisfied demands for both of these activities are $1 + \beta$ times the numbers of unsatisfied demands as listed in Table 5.4. Since $\beta \approx 1$, the numbers of unsatisfied demands are approximately double those which were previously listed. Again we note that these figures do not include unsatisfied demands for copies of titles which are lost, stolen, misplaced, etc.

The final step in this analysis is to determine the relationship between the mean yearly demand rate for titles in subject areas and measures of university community membership with interests in these areas. Specifically, these latter measures include the number of undergraduate students, the number of MBA graduate students and the number of M. S. and Ph.D. graduate students enrolled in courses related to particular subject areas and the number of faculty members with primary research interests and secondary research interests in these subject areas. A distinction is made between MBA and M.S. and Ph.D students because of the differing nature of their respective programs. The MBA program is professionally oriented, whereas the M.S. and Ph.D programs are more academic in nature. This difference in program emphasis undoubtedly results in different uses made of the library by students in the respective programs.

If we let

$X_{i1}$ = the number of undergraduate students enrolled in courses related to subject area i,

$X_{i2}$ = the number of MBA graduate students enrolled in courses related to subject area i,

$X_{i3}$ = the number of M.S. and Ph.D. graduate students enrolled in courses related to subject area i,

$X_{i4}$ = the number of faculty with primary research interests in subject area i,

$X_{i5}$ = the number of faculty with secondary research interests in subject area i,

a least squares regression can be fit to cross-sectional data relating to each subject area for the following model.

$$\overline{D}_i = \beta_o + \beta_1 X_{i1} + \beta_2 X_{i2} + \beta_3 X_{i3} + \beta_4 X_{i4} + \beta_5 X_{i5} + \epsilon_i \qquad (5.9)$$

where $\beta_o$, $\beta_1$, $\beta_2$, $\beta_3$, $\beta_4$, $\beta_5$, are parameters determined by the analysis and $\overline{D}_i$ equals the estimate of the mean yearly demand rate for titles in subject area i as computed by the Circulation Demand Model and as listed in Table 5.4.

Data for the number of faculty members with primary and secondary interests in each subject area were obtained via questionnaire. A copy of the questionnaire used in this effort is included in Appendix F. Data for the number of undergraduate, MBA and M.S. and Ph.D. graduate students enrolled in courses relevant to each subject area were obtained from the Registrar.

Inclusion of all five independent variables in the regression model resulted in a collinearity problem because of the high simple correlation ($r = .80$) between the independent variables $X_{i4}$ and $X_{i2}$, the number of faculty with primary interests and the number of MBA graduate students enrolled in courses in subject area i. A step-wise regression model which sequentially brings in those independent variables which make the greatest reduction in the error sum of squares was performed. In this analysis $X_{i3}$, and $X_{i4}$, were sequentially entered, without encountering collinearity, yielding a multiple $R^2$ of .70. The variable $X_{i5}$ would have entered next. However, its entrance into the model did not significantly reduce the residual sum of squares, at the .05 level. Performance of this analysis resulted in the following regression equation:

$$\overline{D}_i = .1300 + .0149X_{i3} + .0632X_{i4} \qquad (5.10)$$

In this analysis the variable descriptive of the number of M.S. and Ph.D. students enrolled in courses relevant to subject area i, $X_{i3}$, was entered first. This variable explains the greatest proportion of variation in $\overline{D}_i$ ($r^2 = .61$) which is not a surprise. In general, we would expect that Ph.D. and M.S. students, who are involved in a more freely structured program, would make the greatest use of the stacks, whereas MBA students and undergraduates, who pursue a more rigid program, would account for the majority of reserve room uses.

A second regression analysis was performed of $\overline{D}_i$ versus $X_{i1}$, $X_{i2}$, and $X_{i3}$. In this analysis only $X_{i3}$ and $X_{i2}$ were found to be significant, entering the analysis in that order , yielding a multiple $R^2$ of .67. The regression equation was as follows:

$$\overline{D}_i = .2009 + .0173X_{i3} + .0012X_{i2}. \qquad (5.11)$$

The reduction in the proportion of explained variation in utilizing the second model in place of the first is only about .70-.67 = .03. The advantage of using this second model is that the data for the independent variables used in estimating the parameters are readily obtainable from the Registrar as a by-product of the university administrative process. On the other hand, data descriptive of faculty research interests used for estimating parameters in the first model generally require use of a questionnaire. Considering the relatively small loss of precision involved in using the second model and the relative ease in collecting data regarding student enrollment, the second model might be preferable if it were to be implemented on a wide scale basis.

To see how these models can be used to predict changes in the mean demand rate for titles as a function of changes in the composition of the university community, let us suppose that there is to be an increase of 20 students enrolled in Ph.D. courses, 20 MBA students enrolled in courses and one faculty member with research interests all in the same subject area. Using the first model the increase in the mean demand rate as a result of these increases becomes:

$$\Delta\bar{D} = (20)\ (.0149) + (1)\ (.0632) = .3612$$

Using the second model, the increase in the mean demand rate becomes:

$$\Delta\bar{D} = (20)(.0173) + 20(.0012) = .3700$$

At this point we inject a word of caution when using these models to predict future behavior. In the construction of these models cross-sectional data for the mean demand rate and for variables descriptive of the university community were utilized for 39 different subject areas during a particular time period. The parameters of a time series model are estimated using a number of data points descriptive of the activity of a single subject area over time. Estimates of the parameter values for a time series model undoubtedly differ somewhat from the estimates of the parameter values for a cross-sectional model. Further, time series forecasting involves predicting the change in the mean demand rate as a function of changes in the values of the independent variables for a particular subject area. Time series estimates utilizing a cross-sectional model, as we have done in this analysis, undoubtedly contain an element of error.

To the best of our knowledge no comparison of parameters generated by time series analysis and by cross-sectional analysis has been performed in this area. A good deal of research has been reported on in the economics literature regarding the estimation of consumption functions via these two techniques (see, for example, Malinvaud[1]). In some cases, forecasts using a cross-sectional model were found to be very close to the forecasts yielded by the approximate time series model. In other cases the forecasts yielded by these two different models differed significantly. If some time series data are available, the consensus is that these data should be used to obtain a reading on the possible errors which evolve when forecasts are made using a cross-sectional model. If no time series data are available, one should be aware of any possible source of error.

This concludes the analysis of the Circulation Demand Model. In this section a model was developed for predicting the demand rate for the circulation and in-library use of documents as a function of the actual recorded circulation data. Estimates of the number of unsatisfied demands incurred, when a demand is registered for a title for which all copies are out on loan, were computed by subject areas. A model was generated relating the mean demand rate for titles with variables descriptive of the teaching and research programs of the university. A sample calculation was performed indicating how this model can be employed to predict the mean yearly demand rate for titles as a function of teaching and research activities for use in planning library service These estimates of the mean demand rate are used as input data to the models discussed in the ensuing sections.

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[1] Edmond Malinvaud, Statistical Methods of Econometrics, Rand McNally, Chicago, 1966, pp. 124-128.

## Application of The Multiple Copy Selection Model

In the preceding section estimates were computed of the yearly demand rate for titles in each subject area. In this section these estimates are used as input to the Multiple Copy Selection Model which can be solved to aid the library in determining the optimal proportions of duplicate, triplicate, etc. copies to purchase for the new titles selected. The output of this model yields a measure of the marginal expected exposures experienced as a result of the purchase of an additional multiple copy. The criterion employed for purchasing an additional copy of a title is to do so if the expected number of exposures per unit cost experienced by this additional copy exceeds the expected number of exposures per unit cost for the expenditure of funds in alternative programs.

To implement this model, the following tasks were performed:

1. A vector of parameters was estimated relating the mean yearly demand rate for titles in each subject area, $\overline{D}_i$, and the mean yearly demand rate for titles in each subcollection (defined by publication date of titles) of each subject area, $\overline{D}_{it}$. This vector was then used to estimate the mean yearly demand for titles of a subcollection from year to year over the life of the titles.
2. The Multiple Copy Selection Model was solved to yield a matrix of values for $M_{it}$, the expected yearly circulation rate experienced per multiple copy in subject area i, subcollection t. $M_{it}$ is a function of $\overline{D}_{it}$ and m', the proportion of titles in subcollection t, subject area i which comprises the set of titles which experience a higher demand rate.
3. The expected number of circulations experienced by a multiple copy over its life time was determined by summing the expected yearly exposure rate over the life of the title $\sum_t M_{it} = M_i$, where $M_i$ equals the expected number of circu-

lations experienced per multiple copy from the time it is procured until it is discarded.

4. The value of $M_i$ computed in (3) was used to construct a benefit-cost measure descriptive of the expected number of additional exposures per unit cost as a function of an incremental change in the level of m, the proportion of titles for which a multiple copy is purchased.

These tasks are described in succession.

Circulation data were collected for a period of time according to publication date of the circulated titles. From these data estimates were computed for the ratio of the number of titles circulated from each subcollection to the total number of titles circulated, $G_t/G$. A sample of the holdings was drawn to estimate the number of titles held within each subcollection. From these data estimates were computed for the ratio of the number of titles held in each subcollection to all titles held, $N_t/N$. Then for each subcollection the relative circulation rate, $\rho_t = {}^{G_t/G}\!/_{N_t/N}$, was computed as developed in equation (4.30). We observe that

$$\rho_t = \frac{G_t/G}{N_t/N} = \frac{G_t}{N_t}\left(\frac{N}{G}\right) = \overline{G}_t\left(\frac{N}{G}\right) \qquad (5.12)$$

which equals the average circulation rate per subcollection times the constant $^N/_G$. Thus, $\rho_t$ represents a measure of the relative circulation rate of titles in subcollection t These results are shown in Table. 5.5. Since $\rho_t$ represents a measure of the relative circulation rate by subcollections, $\rho_t$ times the mean yearly circulation rate for subject area i, $\overline{G}_i$, yields an estimate of mean yearly circulation rate for those titles in subject area i which are t years old, $\overline{G}_{it}$. The accuracy of these estimates for subject areas depends upon the assumption that the value of the relative measure of popularity or intensity of use of titles as they age, $\rho_t$, is similar across subject areas. This probably is a reasonable assumption for the subject areas within a broad category such as Business.

Estimates of the mean yearly demand rate for titles in subcollection t and subject area i, $\overline{D}_{it}$ are obtained by multiplying the vector of $\rho_t$ values times the scalar $\overline{D}_i$, the mean yearly demand rate for titles in subject area i. These estimates of $\overline{D}_{it}$ are slightly biased downward when $\rho_t > 1$ and upward when $\rho_i < 1$. This bias occurs because the disparity between $\overline{D}_{it}$ and $\overline{G}_{it}$ increases disproportionately as $\overline{D}_{it}$ increases and vice versa. However, as the results are to be aggregated, upward biases will compensate the

### TABLE 5.5

### RATIO OF THE NUMBER OF TITLES CIRCULATED WITHIN EACH SUBCOLLECTION TO ALL TITLES CIRCULATED, $^{G}t/_{G}$, RATIO OF THE NUMBER OF TITLES HELD WITHIN EACH SUBCOLLECTION TO ALL TITLES HELD, $^{N}t/_{N}$, AND THE RELATIVE CIRCULATION RATE, $\rho_t$

| YEAR | $^{G}t/_{G}$ | $^{N}t/_{N}$ | $\rho_t$ |
|---|---|---|---|
| 1969 | .0746 | .0272 | 2.74 |
| 1968 | .1103 | .0410 | 2.69 |
| 1967 | .0929 | .0445 | 2.09 |
| 1966 | .1011 | .0407 | 2.48 |
| 1965 | .0715 | .0455 | 1.57 |
| 1964 | .0633 | .0439 | 1.44 |
| 1963 | .0787 | .0473 | 1.66 |
| 1962 | .0674 | .0440 | 1.53 |
| 1961 | .0633 | .0421 | 1.50 |
| 1960 | .0347 | .0374 | .93 |
| 1959 | .0255 | .0325 | .79 |
| 1958 | .0235 | .0300 | .78 |
| 1957 | .0163 | .0277 | .59 |
| 1956 | .0163 | .0311 | .52 |
| 1955 | .0153 | .0203 | .75 |
| 1954 | .0163 | .0233 | .70 |
| 1953 | .0153 | .0241 | .64 |
| 1952 | .0082 | .0212 | .39 |
| 1951 | .0092 | .0165 | .56 |
| 1950 | .0061 | .0178 | .34 |
| 1949 or before | .0900 | .3419 | .26 |

downward biases, tending to make the overall effect of these biases negligible. Therefore, given an estimate of the mean yearly demand rate for subject area as generated in the preceding section, estimates of the mean yearly demand rate for each subcollection of titles, having the same publication date, can be rendered. With these estimates, the models developed in Chapter IV can be solved to yield estimates of the number of exposures or demands satisfied by the purchase of a duplicate, triplicate, etc. copy. We now proceed to apply this model to determine the proportion of titles for which duplicate copies should be purchased.

To determine estimates of $M_{it}$, the expected mean yearly rate at which a duplicate copy is circulated, the model of equation (4.37) was solved for a vector of demand rates, $\bar{D}_{it}$, and alternative values of $m'$. Since the solution of this expression for $M_{it}$ proves to be complex, numerical integration was performed. Because such a great number of solutions are required for each subject area, this model was solved for a matrix of $\bar{D}_{it}$ and $m'$ values, sufficient to construct a graph depicting a series of curves relating $\bar{D}_{it}$ and $m'$ to $M_{it}$. From Figure 5.1 it is now possible to obtain estimates of $M_i$ for various levels of competency of the decision mechanism. To accomplish this, individual values of $M_{it}$ are determined from

Fig. 5.1--Expected Number of Circulations, $M_{it}$, Experienced by the Duplicate Copy of a Title in the Set of m' Titles in Subject Area i, Subcollection t, Which Experience a Higher Demand Rate, as a Function of the Mean Demand Rate, $\overline{D}_{it}$, Experienced by All Titles in Subject Area i, Subcollection t for $\mu = 52/2.94 = 17.7$ Mean Loan Periods Per Year.

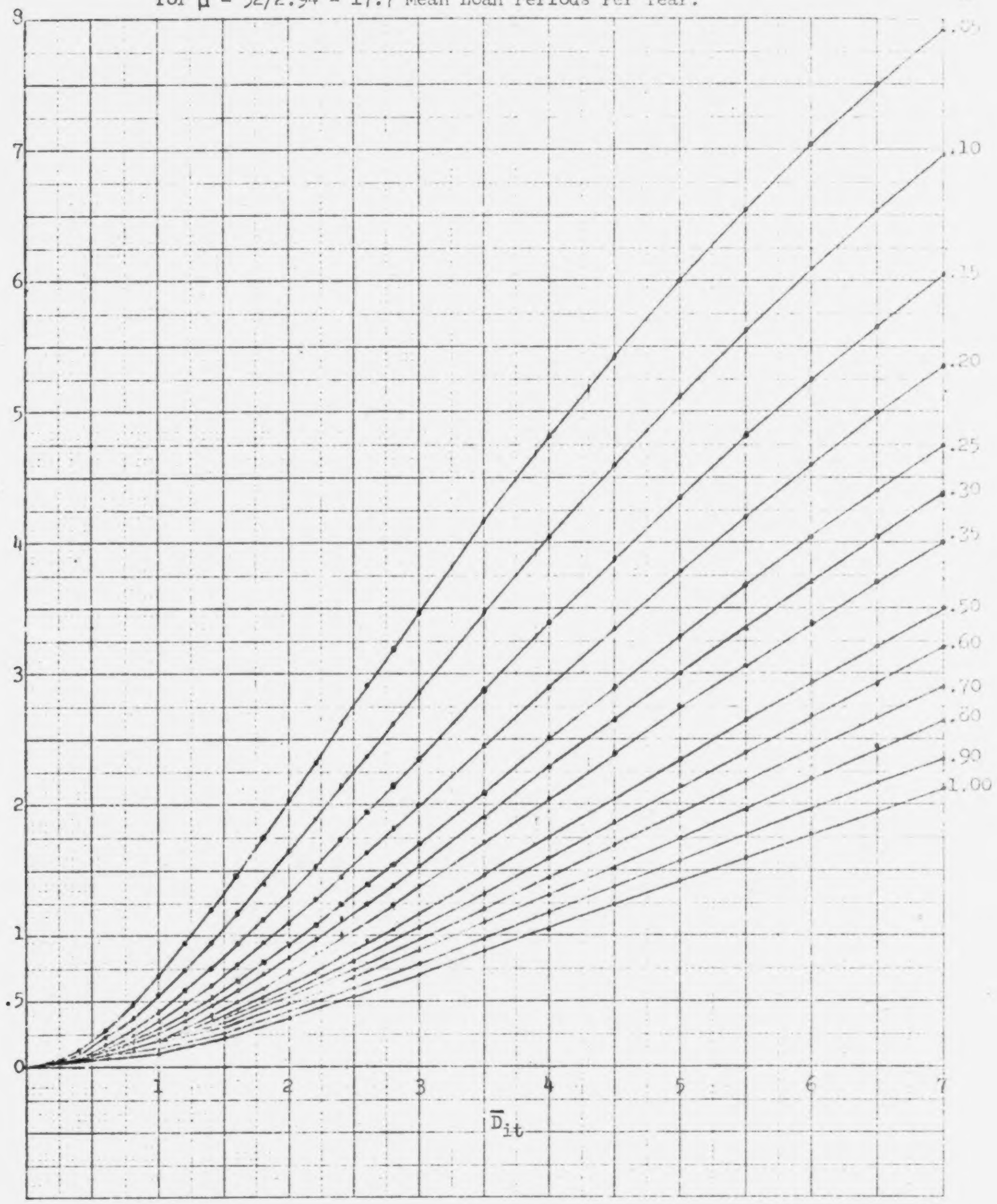

Figure 5.1 corresponding to values of $\overline{D}_{it}$ and a value of m'. The values are then aggregated to obtain $M_i$, where $M_i = \sum_t M_{it}$, the expected number of circulations experienced per duplicate copy over the life of the copy when a duplicate is purchased for one of the set of m' titles which comprise the set of titles which receive a higher demand rate in subject area i.

Given values of $M_i$, the marginal exposure-to-cost ratios, $\Delta M_i/_{\Delta C}$, as specified in equation (4.39), can be computed corresponding to various duplicate copy procurement strategies. When $\Delta M_i/_{\Delta C} = U$, the value of the marginal benefit-to-cost ratio for which total exposures are maximized over all programs, the optimal value of $M_i$ is determined.

As an example of how this model is applied, we determine the optimal strategy for the purchase of duplicate copies for the subject area of Econometrics. From Table 5.4 we observe that $\overline{D}_t$ for Econometrics is 1.567. $\overline{D}$ times the vector of $\rho_t$ values yields a vector of $\overline{D}_t$ values. Corresponding to this vector of $\overline{D}_t$ values are vectors of $M_t$ obtained from Figure 5.1 for various values of m'. These vectors are shown as Table 5.6.

TABLE 5.6

EXPECTED MEAN NUMBER OF YEARLY CIRCULATIONS EXPERIENCED BY A DUPLICATE COPY, $M_t$, AS A FUNCTION OF THE EXPECTED MEAN YEARLY DEMAND RATE FOR TITLES, $\bar{D}_t$, AND THE PROPORTION OF TITLES COMPRISING THE SET OF TITLES EXPERIENCING THE HIGHER DEMAND RATE, m', FOR THE ECONOMETRICS SUBJECT AREA

|  |  |  | m' |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $t$ | $d_t$ | $\bar{D}_t$ | .05 | .10 | .15 | .20 | .25 | .30 | .35 | .40 | .45 | .50 | .55 | .60 | .65 | .70 | .75 | .80 | .85 | .90 | .95 | 1.00 |
| 1 | 2.74 | 4.30 | 5.19 | 4.38 | 3.67 | 3.16 | 2.74 | 2.52 | 2.24 | 2.11 | 2.01 | 1.93 | 1.84 | 1.75 | 1.67 | 1.60 | 1.52 | 1.44 | 1.37 | 1.30 | 1.23 | 1.17 |
| 2 | 2.69 | 4.22 | 5.03 | 4.30 | 3.61 | 3.10 | 2.63 | 2.46 | 2.19 | 2.07 | 1.97 | 1.89 | 1.80 | 1.72 | 1.64 | 1.56 | 1.49 | 1.42 | 1.35 | 1.27 | 1.21 | 1.15 |
| 3 | 2.09 | 3.27 | 3.83 | 3.18 | 2.63 | 2.24 | 1.92 | 1.75 | 1.56 | 1.48 | 1.40 | 1.33 | 1.26 | 1.20 | 1.15 | 1.10 | 1.05 | 1.00 | 0.95 | 0.89 | 0.85 | 0.81 |
| 4 | 2.48 | 3.89 | 4.70 | 3.93 | 3.28 | 2.82 | 2.42 | 2.23 | 1.97 | 1.85 | 1.78 | 1.70 | 1.62 | 1.54 | 1.47 | 1.41 | 1.34 | 1.27 | 1.20 | 1.14 | 1.08 | 1.03 |
| 5 | 1.57 | 2.46 | 2.70 | 2.20 | 1.78 | 1.51 | 1.27 | 1.13 | 1.00 | .95 | .90 | .86 | .82 | .78 | .75 | .73 | .70 | .66 | .62 | .59 | .55 | .52 |
| 6 | 1.44 | 2.26 | 2.38 | 1.95 | 1.58 | 1.32 | 1.13 | 1.00 | .88 | .84 | .80 | .75 | .73 | .70 | .67 | .65 | .61 | .58 | .55 | .52 | .49 | .43 |
| 7 | 1.66 | 2.61 | 2.91 | 2.38 | 1.94 | 1.64 | 1.39 | 1.25 | 1.07 | 1.05 | .98 | .95 | .90 | .86 | .83 | .80 | .76 | .73 | .69 | .64 | .61 | .57 |
| 8 | 1.53 | 2.40 | 2.62 | 2.13 | 1.74 | 1.46 | 1.24 | 1.11 | .95 | .93 | .87 | .84 | .80 | .76 | .74 | .70 | .68 | .64 | .61 | .57 | .53 | .50 |
| 9 | 1.50 | 2.35 | 2.55 | 2.03 | 1.68 | 1.42 | 1.20 | 1.08 | .95 | .90 | .86 | .80 | .78 | .75 | .72 | .69 | .66 | .63 | .58 | .55 | .52 | .49 |
| 10 | .93 | 1.45 | 1.27 | 1.00 | .80 | .67 | .63 | .55 | .47 | .46 | .43 | .41 | .40 | .38 | .36 | .34 | .32 | .31 | .29 | .27 | .23 | .20 |
| 11 | .79 | 1.23 | .94 | .74 | .59 | .49 | .41 | .36 | .31 | .30 | .28 | .27 | .26 | .25 | .23 | .21 | .20 | .19 | .18 | .16 | .15 | .13 |
| 12 | .79 | 1.23 | .94 | .74 | .59 | .49 | .41 | .36 | .31 | .30 | .28 | .27 | .26 | .25 | .23 | .21 | .20 | .19 | .18 | .16 | .15 | .13 |
| 13 | .59 | .92 | .62 | .55 | .45 | .36 | .30 | .28 | .20 | .19 | .18 | .18 | .17 | .16 | .15 | .14 | .13 | .12 | .11 | .09 | .09 | .08 |
| 14 | .52 | .82 | .50 | .40 | .32 | .25 | .22 | .20 | .17 | .16 | .15 | .14 | .13 | .12 | .11 | .11 | .10 | .09 | .09 | .08 | .07 | .07 |
| 15 | .75 | 1.18 | .92 | .72 | .57 | .47 | .40 | .35 | .30 | .29 | .27 | .26 | .25 | .24 | .22 | .20 | .19 | .18 | .17 | .15 | .14 | .12 |
| 16 | .70 | 1.10 | .81 | .65 | .51 | .42 | .35 | .31 | .26 | .26 | .24 | .23 | .22 | .20 | .19 | .18 | .17 | .16 | .15 | .14 | .13 | .12 |
| 17 | .64 | 1.00 | .70 | .55 | .43 | .36 | .30 | .26 | .22 | .21 | .20 | .20 | .19 | .18 | .17 | .16 | .15 | .15 | .14 | .13 | .12 | .11 |
| 18 | .39 | .61 | .27 | .25 | .20 | .16 | .13 | .12 | .10 | .10 | .09 | .08 | .07 | .07 | .07 | .06 | .06 | .05 | .05 | .05 | .04 | .04 |
| 19 | .56 | .87 | .53 | .49 | .36 | .29 | .24 | .22 | .18 | .17 | .16 | .16 | .15 | .14 | .13 | .13 | .12 | .11 | .10 | .09 | .08 | .08 |
| 20 | .34 | .54 | .25 | .23 | .18 | .14 | .11 | .10 | .08 | .08 | .08 | .07 | .07 | .06 | .06 | .06 | .05 | .05 | .05 | .04 | .04 | .04 |
| 21 | .26 | .41 | .26 | .19 | .12 | .10 | .08 | .07 | .06 | .06 | .06 | .05 | .05 | .05 | .05 | .04 | .04 | .04 | .04 | .03 | .03 | .03 |
|  |  | $\sum_t M_t$ = | 39.96 | 33.00 | 27.03 | 22.87 | 19.57 | 17.72 | 15.47 | 14.76 | 13.99 | 13.39 | 12.78 | 12.17 | 11.61 | 11.08 | 10.55 | 10.02 | 9.47 | 8.87 | 8.37 | 7.84 |

where $\bar{D}$ = 1.567 for Econometrics

For illustrative purposes let us assume that the ratio of $m/m'$ is constant and equal to .5. This assumption implies, for example, that if 20 per cent of all the titles are duplicated and 40 per cent of all the titles comprise the set of titles expected to experience a higher rate, then one half of the titles in this latter set are duplicated. Let us further assume that the average cost of purchasing a duplicate, $F_1$, equals $10.00. In addition we assume that the value of U, the marginal exposure-to-cost ratio yielded by the expenditure of additional funds in other programs, equals 1.20. Further discussion regarding the selection of the value of U appears in the conclusion of this chapter. Finally, the values of $M_i$ listed in Table 5.6 refer specifically to expected additional circulations resulting from the purchase of a duplicate copy. The results of the Circulation Demand Model indicate that there is approximately one in-library use exposure for each circulation. Further, if we assume that a title which circulates is worth three exposures, then in all, one expected circulation is worth four expected exposures (1 in-library use exposure plus 3 exposures for the circulation). Thus, in computing the expected number of exposures yielded by the purchase of a duplicate copy of a title, the values of $M_i$ listed in Table 5.6 are multiplied by a factor of 4. Thus we let $W_i = 4$ in equation (4.39).

Using these foregoing assumptions, the optimal strategy for the purchase of duplicate copies for the subject area of Econometrics is now presented. Substitution of the relevant values into equation (4.39) yields:

$\frac{\Delta M_i}{\Delta C}$ for m = .30 to m = .35 is

$$\frac{(4)(11.08)(.35) - (4)(12.17)(.30)}{10(.35 - .30)} = 1.80 > U$$

$\frac{\Delta M_i}{\Delta C}$ for m = .35 to m = .40 is

$$\frac{(4)(10.02)(.40) - (4)(11.08)(.35)}{10(.40 - .35)} = 1.04 < U$$

Considering that U = 1.20, the value of m at which $\Delta M_i / \Delta C = U$ is about equal to .37. Hence if 100 new titles are to be purchased this year, about 37 titles should be duplicated. Similar calculations can be performed for the remaining subject areas and various levels of $m/m'$.

To obtain an idea of the sensitivity of this model, let us suppose that the ratio of $^{m}/_{m'} = .6$ and $F_1$ remains at $10.00 and U remains at 1.20. In this case the optimal value of m is computed to be about equal to .42 as:

$\frac{\Delta M_i}{\Delta C}$ for $m_k = .35$ to $m_k = .40$ equals

$$\frac{(4)(11.51)(.40) - (4)(12.41)(.35)}{10\ (.40 - .35)} = 2.04 > U$$

$\frac{\Delta M_i}{\Delta C}$ for $m_k = .40$ to $m_k = .45$ equals

$$\frac{(4)(10.55)(.45) - (4)(11.51)(.40)}{10(.45 - .40)} = 1.12 < U$$

We observe that this increase in the level of competency in selecting duplicates for those titles which experience the greater demand from $^{m}k/m_k' = .5$ to $.6$ does not radically affect the optimal value of $m_k$. This shift results in a difference of about $\frac{42-37}{42} = 12$ percent in the value specified as the optimal proportion of new titles to duplicate.

In this example exposures were not discounted over time as discussed in Chapter IV. This can easily be accomplished by dividing each $M_{it}$ value by $(1 + r)^t$ prior to aggregating to obtain $M_i$ as discussed in equation (4.38).

This concludes a demonstration of how the Multiple Copy Selection model is applied to the decision for buying duplicate copies of a title. In a similar fashion this analysis can be extended to assist in the decision for purchasing triplicate, etc. copies. In this analysis the value used as an estimate of the mean demand rate, $\overline{D}_i$, was the currently observed value. If the size of the university membership with interests in a particular subject area is expected to change in the near future, more reliable results are obtained using an estimate of $\overline{D}_i$ yielded by the regression model developed in the preceding section.

## Application of Title Selection Model

In this section the model developed to assist in determining the optimal number of new titles to purchase each year is verified and relevant parameters estimated. An example is presented to demonstrate how the model is used in the decision-making process.

The sensitivity of the solution is explored as a function of changes in the values of certain variables in the model. Finally, the validity of certain assumptions made in the construction of the model is investigated.

Initially an analysis was performed to verify that the demand rate for titles decreases exponentially as titles age. This analysis was performed for the aggregate of all subject areas. The data for the dependent variable used in this analysis were the values of $\rho_t$ listed in Table 5.5. We recall that $\rho_t$ = measure of the relative circulation rate of titles in subcollection t. Since $\rho_t$ varies directly as a function of $\bar{G}_t$, the analysis was performed using the data for $\rho_t$.

A least squares regression was performed using the logarithm of the model

$$\rho_t = \Gamma e^{-\psi t} \epsilon_t \qquad (5.13)$$

where t = 1, for the subcollection of 1969 titles, 2 for
the subcollection of 1968 titles, etc.,

$\Gamma$ = scale parameter

$\psi$ = parameter descriptive of obsolescence effect

$\epsilon_t$ = disturbance term

Estimates of the parameter values are as follows:

$$\hat{\Gamma} = 3.25$$

and $\hat{\psi} = .116$

A plot of this exponential function and the actual data values for $\rho_t$ appears as Figure 5.2. Examination of this plot leads us to the conclusion that an exponential function describes the obsolescence of knowledge effect reasonably well. The results of this analysis indicate that the demand rate for titles decreases by a factor of $e^{-.116}$ each year. Although this analysis was performed utilizing data based upon the circulation rate experienced by each subcollection, we would expect similar results if data based upon demand rate had been used. This is so because demand rate and circulation rate vary in approximately the same manner.

Estimates of the number of titles comprising the subcollections of each subject area were made on the basis of a random sample of 200 titles from each subject area. If the subject area contained less that 200 titles, a census of these titles was taken. Details of the sampling procedure utilized are presented in Appendix C.

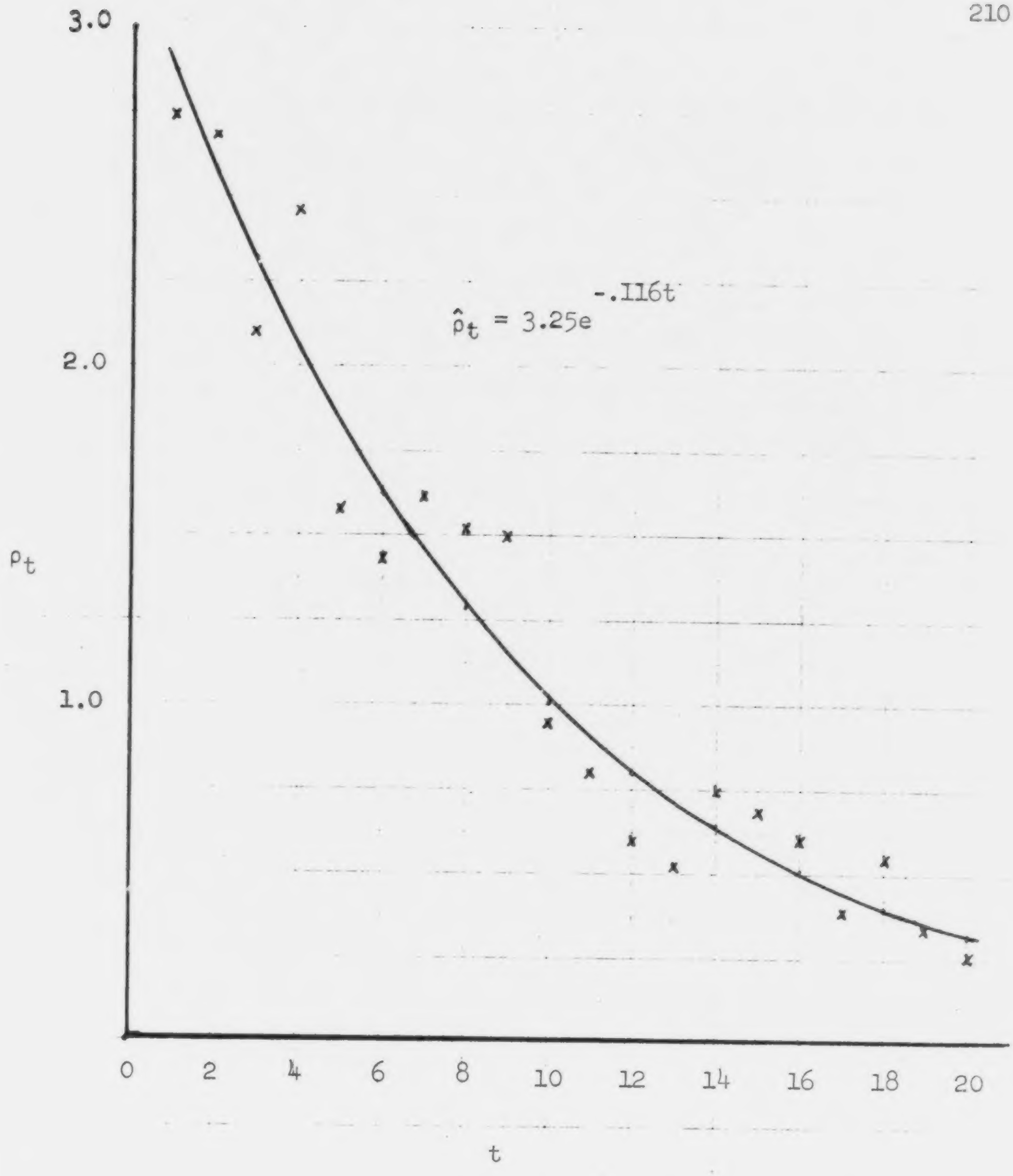

Fig. 5.2--Relationship Between $\rho_t$, the Relative Circulation Rate of Titles in Subcollection t, and t, the Age of the Subcollection Where t = 1 in 1969.

In this model two variables were used descriptive of the research and teaching efforts of the university. They were the number of faculty members with research interests in a particular subject area and the number of M.S. and Ph.D. Degree students enrolled in courses relating to particular subject areas. There two variables were selected because they explained such a high proportion of the variation in the demand rate in the Circulation Demand Model.

Data for the number of circulations were collected according to subject area and publication date of the title during a four-week period. The amount of data collected during this period proved to be insufficient for providing reliable estimates of the number of yearly demands for subcollections of subject areas. To accommodate this deficiency, we found it necessary to combine subject areas largely along subject category lines. In some cases certain categories were excluded from the analysis because of insufficient data, even after the aggregation process. On the other hand, since the Economics subject category comprises such a large part of the entire library collection, it was divided into three categories. The categories resulting from this process, which were included in this analysis, are listed as follows:

## Categories

1. Accounting
2. Economics (I) - Theory, Fiscal Policy and Public Finance, Fiscal and Monetary Theory, History, Systems, Econometrics, Agriculture, Welfare and International
3. Economics (II) - Growth, Development and Planning
4. Economics (III) - Industries, Economic and Social Data
5. Finance
6. Industry
7. Marketing
8. Statistics and Operations Research

During the collection of circulation data an oversight was committed. In this case circulation data pertaining to titles published during and prior to 1959 were aggregated. Because of this factor, useful data were available only for the most recent ten subcollections (1960-1969) in each category.

A subcollection of a category was designated as the unit of association. The numbers of circulations were counted for each subcollection of these categories. Ratios of the number of circulations in each subcollection to all circulations in the relevant categories were then computed. The estimated numbers of demands for each subcollection were determined by apportioning the total number of demands for titles in a category according to these ratios. The estimates of the total number of demands for titles in each category were obtained from the demand rates yielded by the Circulation Demand Model listed in Table 5.4

Thus, given that

$D_{kt}$ = number of circulation demands for titles in category k, subcollection t,

$N_{kt}$ = number of titles in category k, subcollection t,

t = age of subcollection where t = 1 in 1969,

$X_{k1}$ = number of faculty members with interests in category k,

$X_{k2}$ = number of M.S. and Ph.D. Degree students enrolled in courses relating to category k,

a non-linear regression analysis was performed using the following model:

$$D_{kt} = \alpha N_{kt}^{\,\theta} \, e^{-\gamma t} \, (X_{k1} + \delta X_{k2})^{\eta} \, \nu_{kt} \qquad (5.14)$$

for k = 1,...8 and t = 1,...,10, where $\nu_{kt}$ represents a disturbance term.

For 80 data points (8 categories each with 10 subcollections) the following estimates were yielded for the parameters and their respective standard errors.

$\alpha = 7.056$ $\quad S_\alpha = 6.917$
$\theta = .427$ $\quad S_\theta = .075$
$\gamma = .082$ $\quad S_\gamma = .016$

$\delta = 7.127 \qquad S_\delta = 5.457$

$\eta = .152 \qquad S_\eta = .054$

The mean square error for this model was calculated to be 1225. The variance of $D_{kt}$ about its mean was calculated to be 3688. Hence, the amount of variation explained by this model is computed to be

$$\frac{3688 - 1225}{3688} = .67 \text{ or } 67 \text{ percent}$$

Considering the rather high proportion of variation explained by this model, we conclude that the model is a reasonable conceptualization of the user-library interface in this area.

In examining the deviations between actual circulation demand data and the predicted values as yielded by this model, we observe that for some categories these deviations are consistently positive while for others these deviations are consistently negative. This would seem to indicate that there is at least one additional relevant variable, which if included in the model, would lead to a further increase in the proportion of explained variation. The average of the deviations between the observed values and the predicted values for the 10 data points for each category appear in Table 5.7.

TABLE 5.7

AVERAGE NUMBER OF ACTUAL YEARLY CIRCULATION DEMANDS, $\bar{D}_{kt}$, AVERAGE NUMBER OF PREDICTED YEARLY CIRCULATION DEMANDS, $\bar{\hat{D}}_{kt}$, DIFFERENCE BETWEEN $\bar{D}_{kt}$ AND $\bar{\hat{D}}_{kt}$, $\overline{D_{kt}-\hat{D}_{kt}}$, NUMBER OF STANDARD DEVIATIONS AND ESTIMATE OF NEW SCALE PARAMETER, $\alpha_k$.

| Category | $\bar{D}_{kt}$ | $\bar{\hat{D}}_{kt}$ | $\overline{D_{kt}-\hat{D}_{kt}}$ | Number of Standard Deviations | $\alpha_k$ |
|---|---|---|---|---|---|
| Accounting | 31.06 | 54.87 | - 23.81 | - 2.14 | 3.994 |
| Economics (I) | 108.29 | 120.17 | - 11.88 | - 1.07 | 6.358 |
| Economics (II) | 138.39 | 109.11 | + 29.28 | + 2.63 | 8.947 |
| Economics (III) | 70.22 | 89.50 | - 19.28 | - 1.74 | 5.535 |
| Finance | 86.68 | 105.92 | - 19.24 | - 1.73 | 5.774 |
| Industry | 98.99 | 88.74 | + 10.25 | + .92 | 7.867 |
| Marketing | 48.58 | 58.23 | - 9.65 | - .87 | 5.886 |
| Stat. - O.R. | 105.40 | 66.50 | + 38.90 | + 3.50 | 11.183 |

Note:

$$\frac{\text{Standard Error of Estimate}}{\sqrt{n}} = \frac{35}{\sqrt{10}} = 11.11\text{, Number of Standard Deviations} = \frac{\overline{D_{kt}-\hat{D}_{kt}}}{11.11}$$

There is good reason to believe that this observed phenomenon is attributable to differences in the ratios of the number of demands per member for the various categories. We might expect that the number of demands per university community member (demands per capita) would differ from one subject area to another. For example, a student writing a paper in labor relations utilizes many more resources than a student writing a paper in production management. Likewise, the average length of the bibliography for articles published in professional journals relating to certain subject areas which are quantitatively oriented are generally shorter when compared to average length of the bibliography for articles relating to subject areas which are more subjective in nature. These observations point up the fact that there appears to exist a difference in the ratio of documents consulted per capita from area to area. There were not sufficient data points to investigate this phenomenon in the regression relationship of the Circulation Demand Model. Since there are 10 points per category in this analysis, this phenomenon can be explored here.

We observe from Table 5.7 that the average number of actual yearly demands for the category of Economics (I), which contains Economic Growth, Development and Planning, is significantly higher than the average number of yearly demands as estimated by the model. We would predict that in this area the number of demands per member would be higher than average since the literature in this area is generally less analytical in nature. On the other hand, the actual number of demands in the categories of Accounting, Finance and Economics (III), which covers the areas of Industries and Economic and Social Data, are significantly lower than the predicted number of demands. We would expect this as the literature in these areas is generally more quantitative in nature. The exception to this line of reasoning involves the significantly higher than predicted average number of demands experienced by Statistics and Operations Research titles. The rationale for this occurrence is that the quantitative tools and techniques presented in the literature in this category generally apply to all other categories. As a result, a great many demands for titles in the Statistics and Operations Research category are registered by members whose primary interests are in other areas. Thus we would expect the average demand rate for titles in this category to be higher on the average. The categories, which are not significantly different, contain a mix of subject areas for which the literature is both quantitative and non-quantitative in nature.

Estimates of the number of demands experienced by a particular subcollection could be made with greater precision if the model were modified to include the variable descriptive of this varying ratio of demands per member from one category to the next. The simplest way to incorporate this concept into the model is to compute an individualized value of the scale parameter, $\alpha$, for each category. If we let

$\alpha_k$ = scale parameter in the model indicative of the relative number of circulation demands per member relating to category k,

then one method for estimating the value of $\alpha_k$ is:

$$\alpha_k = \left( \frac{\bar{D}_k}{\bar{\hat{D}}_k} \right) (\alpha) \tag{5.15}$$

In this expression it is observed that the new scale parameter associated with category k, $\alpha_k$, is equal to the scale parameter in the original model, $\alpha$, times the ratio of the average actual number of demands divided by the average number of demands as predicted by the original model. Thus if the average actual number of demands exceeds the average number of predicted demands, the value computed for $\alpha_k$ is greater than the value of $\alpha$. Use of $\alpha_k$

in the model results in estimates of the number of demands more in line with actuality.

The estimates of the individualized scale factor associated with each category as computed by this technique appear in the right hand column of Table 5.7.

Incorporation of a differing scale parameter for different categories into the model to account for differing ratios of demands per capita doubtless results in a reduced error mean square. However, a new estimate of the error mean square based upon this modified model and the same data would contain a high degree of bias. An unbiased estimate of the error mean square for this modified model could be obtained by fitting this model, holding the values of $\alpha_k$ constant, to a new set of circulation demand data.

Use of an individual value of $\alpha_k$ for each category necessitates certain changes in the models developed in Chapter IV. In the expression for determining the optimal number of new titles to purchase as of function of the parameters and the marginal exposure-to-cost ratio for the expenditure of funds in other programs, equation 4.53, $\alpha_k$ is substituted for $\alpha$. The expression for determining the optimal number of new titles to purchase for category k relative to the number of new titles purchased in category $\alpha$, equation (4.56), becomes

$$N_{kt}^{o} = N_{jt}^{o} \left( \left( \frac{\alpha_j}{\alpha_k} \right) \left( \frac{X_{1j} + \delta X_{2j}}{X_{1k} + \delta X_{2k}} \right)^{\eta} \right)^{1/\theta - 1} \qquad (5.16)$$

To see how this model is applied in the decision-making process, we determined the optimal number of new titles to purchase in the Accounting Category. Assuming that

$U = 1.20$ (marginal exposure-to-cost ratio)
$X_1 = 15$ (present number of faculty)
$X_2 = 69$ (present number of M.S. and Ph.D. Degree students)
$F_2 = \$20$ (estimated cost of adding an additional title)

then using the adjusted value for $\alpha_k$, 3.994, in addition to the other parameter estimates, equation (4.53) can be solved to determine the optimal number of new titles to purchase.

One final adjustment is required, prior to the solution of this expression, to obtain the optimal value of $N_{k1}^{o}$. The initial model was developed in terms of the number of demands for circulation use of titles in each subcollection. If we assume that each circulation use is worth four exposures, as we did in the computations for Multiple Copy Selection Model (one in-library use plus three exposures for one circulation use), and if we assume that an exposure to an additional copy is worth the same as an exposure to an additional title, then the scale factor in the model, $\alpha_k$, must be multiplied by a factor of 4. In this manner, results are obtained which are consistent with maximizing exposures over both programs. Thus we let $W_i$ equal 4 in equation (4.53).

Given the preceding values for the parameters and variables, solution of equation (4.53) to determine the optimal number of new accounting titles yields:

$$N^{\circ} = \left( \frac{(4)(.427)(3.994)(15 + (7.127)(69))}{(20)(e^{.082} - 1)(1.20)}^{.152} \right)^{\frac{1}{1 - .427}}$$

$$N^{\circ} = 37.6$$

Similarly, we determine the number of Accounting titles to purchase relative to the number of titles to purchase in another category, Finance for example, as follows. Let us assume that for Finance

$X_1 = 26$ (present number of faculty members)

$X_2 = 67$ (present number of M.S. and Ph.D. Degree students)

Then, using equation (5.16), the optimal number of titles to purchase in Accounting, relative to Finance, is computed to be

$$T^{o}_{(ACCT.)} = T^{o}_{(FIN.)} \left(\frac{3.994}{5.774}\right) {\left(\frac{26 + (7.127)(67)}{15 + (7.127)(69)}\right)^{.152}}^{\frac{1}{.427-1}}$$

$$= T^{o}_{(FIN.)} (.534)$$

Thus, on the average, the purchase of one Accounting title for each .534 Finance title will result in an effective allocation of funds for the purchase of new titles between these categories.

To obtain a feeling of the sensitivity of this model, the optimal number of Accounting titles was computed for a 50 per cent increase in the number of faculty and M.S. and Ph.D. students enrolled in Accounting courses. For this example

$$X_{1k} = (1.5)\ 15 = 23$$

$$\text{and } X_{2k} = (1.5)\ 69 = 104$$

Solution of the expression for U = 1.20 yields:

$$T^{o} = \left[ \frac{(4)(.427)(3.994)(23 + (7.127)(69))^{.152}}{(20)(e^{.082} - 1)(1.20)} \right]^{\frac{1}{1-.427}}$$

$$T^{o} = 43.1$$

We observe that a 50 per cent increase in membership results in a $\frac{43.1-37.6}{43.1}$ = 13 per cent increase in the number of new titles to purchase as specified by this model. This slight increase in the $T^{o}$ as a result of a 50 per cent increase in $X_{1k}$ and $X_{2k}$ might seem surprisingly low. However, other research findings indicate that a relatively small proportion of titles account for the vast majority of demands.[2] Further, most of the additional demands generated by an increase in the membership will be for this relatively small proportion of titles. Only a few demands will be registered for titles which are peripheral to the collection. Thus it is worthwhile, in terms of increased exposures per cost, to add only a few additional titles. On the other hand, we would expect a significant increase in the number of multiple copies purchased, as specified by the Multiple Copy Selection Model, in response to an increase in membership.

An increase in the marginal benefit-cost ratio, U, from 1.20 to 1.40 results in a decrease in the optimal number of new accounting titles to purchase from 37.6 to 30.0. A decrease in the marginal benefit-cost ratio, U, from 1.20 to 1.00 results in an increase in the optimal number of new Accounting titles to purchase from 37.6 to 50.9. The directions of these changes are predictable. For example, if the marginal benefit-to-cost ratio increases for the expenditure of funds in other programs we would expect to achieve increased effectiveness by transferring funds from the purchase of new Accounting titles to these other programs. The converse is true if the value of U for other programs were to decrease. Further, we expect that the number of exposures experienced by the purchase of an additional title decreases as the number of titles increases. Thus the absolute value of the marginal exposure-to-cost ratio for purchasing one additional title is necessarily less than the marginal exposure-to-cost ratio for purchasing one less title. This relationship accounts for the disproportionately higher number of titles purchased associated with a .20 decrease in U relative to the number of titles purchased associated with a .20 increase in U.

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[2] Richard Trueswell, "A Quantitative Measure of User Circulation Requirements and Its Possible Effects on Stack Thinning", American Documentation 16:20-25 (January 1965).

In this analysis cross-sectional data were utilized descriptive of circulation demands for titles in a series of subcollections during a particular period of time to provide estimates for relevant parameters of the model. The solution of the model was then used to predict demands for the use of titles in subcollections over time. As indicated in a preceding section, the employment of this technique, in certain economic models, sometimes yields "good" estimates and other times yields "poor" estimates of the relevant parameters. In general "good" estimates are yielded by this technique if there are no factors present which affect the demands for a subcollection over time differently than the demands for a series of subcollections during a particular period of time. There is no reason to believe that such a factor is present in this model. None-the-less if time series data were available, it would be desirable to validate this conclusion.

There is reason to believe that the values of the parameters of the model will alter as the results of this model are implemented. Data for the number of titles held in each subcollection, utilized in estimating the parameters of the model, are descriptive of past title selection strategy. Given a change in this strategy, we might expect parameter values to change. In this regard we recommend a periodic review program to ascertain any changes in the effectiveness or parameters of this model. In general this type of program represents sound managerial practice, not only for this model, but for all decision models.

Finally, we again caution that the outputs of this model should serve as a guide in determining the optimal number of new titles to purchase in each category. In fact, it might be the case that the specified number of new titles to purchase exceeds the number of new title publications. As with the outputs of any model, the results of this model provide aids for making improved decisions and should not be implemented blindly.

## Reserve Model Application

In this section the Reserve Model developed in Chapter IV is applied to the reserve service operation of the Lippincott Library. In the analysis the parameters for the regression model which relates demand for the use of titles on reserve with number of students enrolled in courses for which the titles have been assigned. Using this demand function and the marginal exposure-to-cost ratio developed in Chapter IV, sample computations are performed to demonstrate how this model is utilized to yield specifications for the optimal number of copies of titles to place on reserve. Finally the sensitivity of the model is examined and limitations of the model discussed.

During a two-week period, use data were recorded for a sampling of titles which were actively on reserve in Lippincott Library and not on reserve at any other departmental library during the spring semester of 1970. The term "actively on reserve" refers to those titles in the reserve collection which a professor has designated as assigned reading material in connection with a course offered during the indicated spring semester. Other titles in the reserve collection were actively on reserve only during the fall semester, while other titles were actively on reserve during the previous spring semester, but not during the spring semester of 1970. In some cases copies of titles on reserve in Lippincott Library are also placed on reserve at other departmental libraries. As use data were not collected for these copies, these titles were eliminated from the analysis.

For each title in the sample, counts were obtained of the number of students enrolled in the course(s) for which the title was designated as assigned reading material. Sometimes the same title was used for more than one course. In this case the count included the total number of students enrolled in all courses for which the title was designated as assigned reading.

An estimate of the mean loan period for titles on "closed reserve" was obtained by noting the time of withdrawal and return for a sampling of titles. Books on "closed reserve" are shelved in a restricted area behind the reserve desk, must be signed for by the user, and should be returned to the reserve desk within two hours. If another user desires a copy and all copies of the title are out on loan, a user violating this two hour limit is subject to a fine. On the other hand if no other users are waiting for a copy, this two hour limit is not enforced. For the titles on closed reserve, the estimated mean use period for a sample of 50 titles was 2.1 hours.

A sampling of titles on "open reserve" yielded an estimate of the mean use period of 5.1 hours. Documents on "open reserve" are readily accessible to users, may be used within the library for an unlimited length of time, and are generally not returned to the shelves by users. The difference in the mean use period, for titles controlled in these different manners, is primarily attributed to the delay experienced by books on open reserve from the time a user leaves the title on a study table to the time a library attendant collects and reshelves it. The actual mean time that a title is actively being used by a patron is similar in each case.

To obtain use data for titles on open reserve, counts of the number of copies of a title not on the shelf were made at random times. The average number of copies of a title "off the shelf" during the two-week period times the weekly use rate yields an estimate of the average number of uses for that title during a one-week period (as discussed in Chapter IV, equation(4.69)). For example, let us suppose that on the average of 1.5 copies of a title were "off the shelf" during the sampling period. As the library is open approximately 85 hours a week and as the mean use period was estimated to be 5.1 hours, then $\mu = 85/5.1 = 16.8$ use periods per week. The number of times that this title was used per week is estimated to be

$$(f)(\mu) = (1.5)(16.8) = 25.2 \text{ uses per week}$$

Data for the number of uses of a title in the sample on "closed reserve" were obtained by counting the number of uses as recorded on the signout card(s) of all copies of the title during the sampling period.

Initially, a least squares linear regression analysis was performed on these data with $n = 153$ titles. A title was the unit of association with the average number of uses per week as the dependent variable and the number of students enrolled in courses for which the title had been designated for assigned reading as the independent variable. This analysis yielded a coefficient of determination ($r^2$) of .17, which is just significant at the .05 level. As this analysis yielded a relationship for which only 17 per cent of the variation in the dependent variable could be explained by the independent variable, a refinement of the data descriptive of the number of students enrolled in courses was attempted.

A questionnaire was administered for each title in the sample to the professor in charge of the course for which the title was assigned reading. The professor was asked to indicate whether a single section or more than one section of the title was _required_ reading for some or all students enrolled in the course or whether the title was assigned as _background_ reading. Of the 153 titles in the original sample, questionnaires were completed for 104 titles. Of the 104 titles, 65 were cited as _required_ reading for students, 29 were cited as _background_ reading and 10 were cited as not being used at all. A copy of the questionnaire is included in Appendix F. Because of the sparsity of data, the responses in categories 1, 2, 3, and 4 of the questionnaire were aggregated and termed "required reading" for students in this analysis.

A least squares linear regression analysis was performed for the 29 titles designated for background reading. This regression of the number of uses of each title versus the number of students enrolled in courses assigned to read the title as background reading yielded no significant relationships. A perusal of the data revealed that a majority of these titles experienced zero uses during the sampling period. Considering the meager use rate experienced by these titles, this would seem to indicate that the placing of a single copy of these titles on reserve is sufficient to meet user needs.

A least squares linear regression analysis of 64 titles[3] designated as required reading by students yielded a significant relationship ($r = .76$) with

$$Y = -2.07 + .148X$$

$$S_{Y.X} = 5.2$$

where $Y$ = average number of reserve uses per week of a given title

$X$ = number of students enrolled in courses for which the title was assigned as required reading

The linear model provided a good fit to the data with 57 per cent of the variation in the Y variable explained by the X variable.

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[3] Note: One extreme data point (0 uses, 180 students) was excluded from this analysis. Inclusion of this point would result in an $r^2 = .47$. The rationale for excluding this point was based upon a belief that this was not a representative point. There are two reasons for this belief. The first is that there is a likelihood that the use data for this particular title may have been discarded inadvertently. The second reason resulted from a discussion with the professor in charge of the course. He indicated that the title consists of a collection of reprinted journal articles. Students were expected to read one article which was also available in a bound volume of the journal. In addition the professor indicated that the article was not critical to the conduct of the course. In fact, the article was discontinued as a required reading for the course after the Spring 1970 semester. In view of these reasons, this data point was not included in the final analysis.

A more significant relationship might result (one which explains a higher percentage of the variation in Y) if titles which were designated as course textbooks were treated separately. Most students purchase a personal copy of a required textbook. Because of this factor, titles on reserve which are also textbooks, probably experience fewer demands on the average than other books. Unfortunately, when the questionnaire was administered, professors were not asked to indicate which of the titles designated as required reading material might also be assigned textbooks. Inclusion of this item in the questionnaire in future studies is recommended.

Undoubtedly, a further reduction in the unexplained variation would occur if use data had been collected for each title in the sample over the entire semester and averaged to obtain the weekly use rate. In general we would expect that the intensity of use of a particular title during a particular week is somewhat dependent upon the time sequence in which topics are covered in the course. For example, if a title is not relevant to the conduct of the course until late in the semester, we would expect that the title would receive little use in the early part of the semester. This latter situation undoubtedly accounts, in part, for the rather high proportion of titles in the sample (about 1/4) which experienced zero demands during the two-week period in which data were collected.

Estimates of the average weekly demand rate, based on data collected in the latter part of the semester, probably are biased in an upward direction for titles relevant to a course in the latter part of the semester and biased downward for titles relevant to a course in the early part of the semester. However, when these data are used in the regression analysis, we would expect that in the aggregation process, negative biases would tend to cancel positive biases resulting in reliable estimates of the relevant parameters.

Using this regression model, forecasts can be made regarding the expected demand rate for the titles on reserve designated as required reading material. These estimates can then be used, in conjunction with the benefit-cost ratios and criterion developed in Chapter IV, to determine the optimal number of copies of a title to place on reserve.

To see how this is accomplished, let us assume that a professor indicates that a title is to be assigned for required reading in a course for which the enrollment is 50 students. Using the regression model, the predicted average weekly demand rate for this title is:

$$Y = -2.07 + .148(50) = 5.3 \text{ demands/week}$$

If we assume that this title is to be used in conjunction with this course for 3 years during 15 weeks of one semester, then the value of $\lambda m$ of the model of equation (4.68), the total expected number of demands for use of the title, equals $(3)(15)(5.3) = 238.5$. Let us assume that the cost of purchasing an additional copy of a title is \$10.00. Finally, let us assume the average user is willing to wait no longer than one hour to obtain a copy. Hence, $\tau = 1$ hour $= {}^1/85$ week (as the library is open 85 hours per week). If a title is to be placed on "closed reserve", the mean use period, $^1/\mu$ is 2.1 hours or $^{2.1}/85$ weeks. Hence, given

$$\lambda = 5.3 \text{ demands per week}$$

$$\mu = {}^{85}/2.1 \text{ weeks}$$

and

$$\tau = {}^1/85 \text{ weeks}$$

$P(t > {}^1/85 \mid c = 1)$, $P(t > {}^1/85 = 2)$, etc. can be computed. As indicated previously, Molina[4] has prepared extensive charts for determining these probabilities. These charts appear as Appendix G of this dissertation. In using these charts,

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[4] Edward Molina, "Applications of the Theory of Probability to Trunking Problems", The Bell Telephone Technical Journal, 6:461-494 (1927).

$$\frac{a}{c} = \frac{\lambda}{\mu c} \quad \text{and} \quad \frac{t}{h} = \frac{\tau}{1/_{\mu}} = \tau\mu$$

Thus in this example we observe that

$$\text{for } \frac{\lambda}{\mu c} = \frac{5.3}{(85/2.1)c} = \frac{.13}{c} \text{ and } \tau\mu = (\frac{1}{85})(\frac{85}{2.1}) = .48,$$

$$P(t > {}^1/85 \mid c = 1) = .085$$

$$P(t > {}^1/85 \mid c = 2) = .003$$

$$P(t > {}^1/85 \mid c = 3) = .000^+$$

The marginal benefit-to-cost ratio in providing 2 copies as compared to one, using equation (4.68) is

$$\frac{238.5(.085-.003)}{\$10} = \frac{19.5}{10} = 1.95 \text{ exposures per dollar}$$

The marginal benefit-to-cost ratio in providing 3 copies as compared to two is

$$\frac{238.5(.003-000)}{\$10} = \frac{.72}{10} = .07 \text{ exposures per dollar}$$

If the marginal benefit-to-cost ratio for investing incremental funds in other activities is U = 1.20, then we would conclude that two copies of the title in question should be placed on reserve as $\frac{\Delta E}{\Delta C} > 1.20$ for two copies but $\frac{\Delta E}{\Delta C} < 1.20$ for 3 copies. In this case we would expect to lose (.003)(238.5) = .7 exposures by not providing the third copy.

If, on the other hand, this title is to be placed on "open reserve," $\mu = {}^{85}/5.1$, then

$$\frac{\lambda}{\mu c} = \frac{5.3}{(85/5.1)(c)} = \frac{.32}{c} \quad \text{and} \quad \tau\,\mu = \left(\frac{1}{85}\right)\left(\frac{85}{5.1}\right) = .20$$

and $P(t > 1 \mid c = 1) = .270$

$P(t > 1 \mid c = 2) = .035$

$P(t > 1 \mid c = 3) = .003$

$P(t > 1 \mid c = 4) = .000^{+}$

Then the relevant benefit-cost ratios are computed to be

$$238.5\,\frac{(.270 - .035)}{\$10} = 5.64 \text{ exposures per dollar}$$

for c = 2 copies

and $$238.5\,\frac{(.035 - .003)}{\$10} = .76 \text{ exposures per dollar}$$

for c = 3 copies

as $\Delta E/\Delta C > 1.20$ for $c = 2$ but $\Delta E/\Delta C < 1.20$ for $c = 3$, the optimal decision is also to purchase 2 copies of the title if it is to be placed on "open reserve". In this case we would expect to lose $(.003)(238.6) = 7.6$ exposures by not placing the third copy on reserve.

In a similar fashion, this model can be utilized to compute the benefit-cost ratio for procuring an additional copy of a title for reserve ("open" or "closed") for appropriate values of $\lambda$, $\mu$, and $\tau$. For values of $\tau$ and $\mu$ and a relationship between students and demand rate, $\lambda$, a schedule can be constructed indicating the optimal number of copies to place on reserve as a function of the number of students enrolled in courses assigned to read particular titles.

Some cautions and limitations relating to the use of this model are discussed below.

In the preceding example the conditional mean of the regression model was used as a point estimate of the average weekly demand rate for the use of a title which 50 students were assigned to read. This represents an estimate of the mean of the average weekly demand rate. Individual values of the average weekly demand rate vary about this mean. In fact, since the standard error of the estimate equals 5.2, it can be stated that approximately 90 per cent of the titles, for which 50 persons have been assigned reading, experience an average weekly demand rate of less than

$$5.3 + (1.28)(5.2) = 12.0 \text{ demands per week}$$

To determine the optimum number of copies of a title to place on "closed reserve," using the previously developed criterion, for a title which experiences an average weekly demand rate of 12.0, the following terms are computed:

$$\frac{\lambda}{\mu c} = \frac{12.0}{(\frac{85}{2.1})(c)} = \frac{.29}{c}, \quad \tau\mu = (\frac{1}{85})(\frac{85}{2.1}) = .48$$

$$m\lambda = (3)(15)(12.0) = 540$$

Then, from the charts in Appendix G.

$$P(t > \frac{1}{85} \mid c = 1) = .200$$

$$P(t > \frac{1}{85} \mid c = 2) = .015$$

$$P(t > \frac{1}{85} \mid c = 3) = .001$$

$$P(t > \frac{1}{85} \mid c = 4) = .000^{+}$$

The relevant benefit-cost ratios are

$$\text{for } c = 2, \frac{540(.200-.015)}{\$10} = 8.29 \text{ exposures per dollar}$$

$$\text{for } c = 3, \frac{540(.015-.001)}{\$10} = .810 \text{ exposures per dollar}$$

In this case the decision remains to purchase 2 copies, since the incremental benefit-cost ratio for adding the second copy is greater than U (8.29 > 1.20), and the incremental benefit-cost ratio for adding the third copy is less than U (.81 < 1.20).

This analysis provides an idea of the sensitivity of this model. For the case in which 50 students are assigned to read a title, use of the upper 90 per cent confidence limit as the estimate for the average weekly demand rate as opposed to the conditional mean, leads to the identical conclusion regarding the number of copies of the title which should be placed on reserve. However, we notice that the critical marginal benefit-cost ratio increased from .07 to .81. In this example, if the average weekly demand rate is actually 12.0, the placing of the third copy on reserve would satisfy an additional 540(.015-.001) = 8.1 demands of the 540 demands experienced by this title over a three year period. From this analysis we conclude that the solution of the model might lead to the purchase of an additional copy for reserve in some situations although this was not the case here.

This variation of the average weekly demand rate for an individual title about the conditional mean poses a problem. In Chapter IV we indicated that if the standard error of the estimate is small, use of the conditional mean of the regression model to predict individual average weekly demand rates would lead to good approximate results. However, in this analysis the standard error of the estimate was rather large ($S_{Y.X} = 5.2$). Use of a higher limit ( e.g. 90 per cent confidence limit) as an estimate of the average weekly demand rate for an individual title insures that on the average a greater number of demands is satisfied with less waiting time. However, use of this value in the model may result in a decision to place an excessive number of copies of a title on reserve for some titles which leads to an ineffective allocation of resources. The solution to this dilemma lies in designing the analysis in such a way that the standard error of the estimate is reduced. Some techniques have already been discussed for reducing this error. One proposal is to compute the value for the average weekly use rate on the basis of use data for a title collected over an entire semester rather than on the basis of use data collected for a two-week period. This should have the effect of eliminating high and low extreme points, thereby reducing the variation. Another possible source of extreme observations results from the inclusion of assigned textbooks in the sample of reserve titles. Exclusion of these textbooks might yield a reduction in the standard error of the estimate. Finally professors should be queried to indicate how a particular title is to be used in a course. This would provide an indication of different intensities of use experienced by a title according to the manner in which it was assigned. The five categories appearing on the questionnaire in Appendix F represent a good basis for ascertaining different intensities of title use. In this study sufficient data were not collected to perform separate analysis relating average weekly use rate with the number of students enrolled in a course assigned to read the title for each of the five categories. However, it is our belief that inclusion of these concepts into the analysis will have the effect of reducing the standard error of the estimate to the point where reliable approximations will result when the conditional mean of the regression model is used as an estimate for the average weekly demand rate for an individual title.

Previously we indicated that for the aggregate of titles, there did not appear to be a pronounced seasonal pattern although number of uses was slightly higher during the latter part of the semester. This lack of a pronounced seasonal pattern for the aggregate of titles does not imply the same holds true for a single title. If there is a marked seasonal pattern for individual titles, the optimal number of copies to place on reserve, as determined by an assumption of a constant average weekly demand rate, might contain an element of bias. In this case if the bias is considered significant, steps must be taken to incorporate seasonal variation into the model similar to what was done in the Circulation Demand Model.

The selection of 3 years as the average period of time for which a title is on reserve was estimated empirically from a sampling of titles. This was accomplished by noting the length of time a title had been on reserve for those titles on reserve in spring semester of 1969 but not on reserve during the spring semester of 1970. This is probably a relatively accurate estimate of the true mean time.

In this example the selection of one hour as the length of time the average user was willing to wait for a copy of a title was somewhat arbitrary and based upon personal experience. Not much is known concerning the behavior of students when they are unsuccessful in obtaining a desired book immediately. This area is deserving of further research. It might be that demand for a reserve title is relatively inelastic with respect to waiting time with students willing to wait extended periods of time to use a copy of a title. In this case the selection of the value for $\tau$ might include a consideration of the time spent by students waiting for a copy of the title.

In the development of this model, an assumption was made that the relationship between average weekly demand rate and students in courses could be constructed based upon actual average weekly use rate data. For this assumption to hold, there must be few instances of unsatisfied demands for the titles in the sample during the sampling period. In this study rarely were all copies of a reserve title being used at the same time. If this had not been so, it would have been necessary to obtain data directly on the requests for a copy rather than on the use of a copy.

Finally, the output of this model, in terms of a prescribed number of copies of a title to place on reserve, should be used for guidance in the selection of reserve copies. For example, if it is obvious that a title will be used for a period of time less than 3 years, the full complement of copies should not be purchased. An example of this might occur when a professor requests that the most recent volume of a serial be placed on reserve each year. Hence as with the output of any model, the output of this model requires evaluation by a competent decision maker.

## Summary and Conclusions

In this chapter the analytical decision models developed in Chapter IV were tested. We concluded that each of the models represents a sufficiently valid conceptualization of actuality to be of value in the decision-making and planning process. In this regard parameters were estimated and example solutions determined relevant to the operations of the Lippincott Library. Finally, the sensitivity and limitations of these models were explored and discussed.

Initially, the components of the Circulation Demand Model were tested and relevant parameters estimated. Estimates of the mean demand rate for circulation use of titles were computed as a function of actual circulation use data of titles in the subject areas maintained by Lippincott Library. The form of the relationships between circulation use and in-library use was verified indicating that there are approximately one in-library use for each circulation use for this library. Finally, parameters were estimated for the model which relates the mean demand rate for titles with variables descriptive of the research and teaching efforts of the university. In this regard the variables which explained the greatest amount of variation in the mean demand rate were the number of faculty with primary interests in a subject area and the number of M.S. and Ph.D. Degree students enrolled in courses relating to a subject area.

Estimates were computed for the expected number of additional uses experienced by the purchase of a duplicate copy for use in the Multiple Copy Selection Model. An example was performed to demonstrate the application of this model for determining the optimal proportion of new titles to duplicate for one subject area.

Values were estimated for the parameters of the Title Selection Model. The model was modified to account for a variable which had not originally been included. We surmised that this variable provided a measure of the differing ratios of demands per capita attributed to differences in the subject matter of the various subject categories. An example of the application of the model was performed to determine the optimal number of new titles to purchase for a particular subject category.

Finally, an example of the application of the Reserve Service Model was demonstrated. Initially, a regression analysis was performed to obtain estimates of the parameters relating the demand for the use of a reserve title with the number of students enrolled in courses for which the title was assigned as required reading. Estimates of the mean expected demand rate rendered by this model were then utilized in the computation of a measure of marginal exposure-to-cost for a particular example. Solution of an expression which equates this latter measure with a value of the library-wide marginal exposure-to-cost ratio, yielded a specification of the optimal number of copies of a title to place on reserve.

At this juncture at least one major question remains unanswered. This relates to the means for determining the individual library's value of U. In our analysis U represented the value of the marginal exposure-to-cost ratio for which resources were allocated in the most effective manner on a library-wide basis. A determination of the value of U is vital to the solution of the decision models developed in this dissertation.

We venture to say that, at present, precious few university libraries have a notion for their value of U. This result does not lead to the conclusion that the foregoing decision models are inoperable. This merely indicates that additional effort must be expended to arrive at an effective scheme for the allocation of resources. However, the results of these additional efforts can be extremely useful in the negotiation process with the university for determining the library's budget.

Using decision models of the type developed in this research, it is possible to determine a schedule of the required spending levels associated with alternative values of marginal exposure-to-cost. This can be accomplished by solving the foregoing models for various values of the marginal benefit-to-cost ratio and determining the funding requirements for the purchase of documents associated with the resulting solutions. In this manner, the library administrator will be in the enviable position of knowing exactly how an increase or decrease in funds will affect the number of document exposures. This schedule of marginal exposure-to-cost ratios will doubtless be an important factor in the negotiation process for substantiating and justifying requests for additional funds. This technique also presents the university administrators with a tangible measure for determining an effective allocation of resources between the library and other university programs. Once the final figure for the library's budget has been determined it is then possible to consult this schedule to determine the corresponding marginal exposure-to-cost ratio. This resulting marginal exposure-to-cost ratio becomes the value of U for that particular library.

This concludes the chapter designed to verify and apply the previously developed models. In the succeeding chapter we discuss the implications of these decision models in the development of a management information system.

# CHAPTER VI

# DIRECTIONS FOR THE DEVELOPMENT OF A STATISTICAL INFORMATION SYSTEM

## Introduction

An effective management system requires some form of Statistical Information System (SIS) which collects and synthesizes information relevant to a problem and which conveys this information to the administrator(s) responsible for its solution. This is not meant to constitute an invitation for the library administrator to create an information system by immediately embarking upon an all out effort to gather and process a great assortment of information. Management in many sectors of the economy is experiencing disillusionment and disenchantment with the results promised from the employment of such information systems. In many cases, the impressive stream of information flowing from these systems has only served to confuse and overwhelm the decision maker at an increase in costs. Ida Hoos commenting on the assumption that administrators would make better plans and improved decisions if they had more information states:

> We now know that they cannot look to information systems as designed and merchandised to help them much. In fact, there is more likelihood that they will be inundated by an overabundance of data that will impede their efforts to understand problems in their true and dynamic dimensions. Data processed because they are machine-processable provide a shaky foundation, indeed, for community planning.[1]

It seems that for many organizations these highly touted systems, spewing out massive bits of information in a reduced time frame at increased costs have provided little in the way of contributing to the solution of management problems. In fact, these systems may prove to be more detrimental than beneficial in the management process.

On the other hand, at the other extreme, insufficient information leads to inadequate planning and ineffective decision making. This condition is likewise not a very desirable state. Thus it would seem that the composition of a library's SIS lies somewhere between these extremes.

To provide assistance in specifying an effective SIS, we focus on the concept of the value of information to the library. In this context information possesses value only if it can be used in the management process. Information which is not actively used in the management process contributes nothing, except possibly confusion and increased costs.

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[1] Ida Hoos, "Information Systems and Public Planning." Management Science, Vol. 17, June 1971, P. B-669.

Information in this latter category possesses zero or possibly negative value. Thus, the information elements included in a library's SIS should be scrutinized according to their value in the management process. It is in this spirit that we approach the concept of a library SIS. In this chapter we discuss the concept of the value of information in the library management process. Then, using this concept we outline directions for the development of a university library SIS.

Finally, some researchers find it necessary to make a distinction between the concept of data and information. In this regard information is considered to be data which has been processed or analyzed in some fashion. We do not find this distinction necessary. Hence, in this dissertation the terms "data" and "information" are used synonymously.

## Value of Information

In the context of the library management process, information would seem to possess value when used for decision-making, planning or persuasive purposes. In this dissertation we have focused primarily on the decision-making and planning aspects of management. Little has been discussed regarding the persuasive or influencing aspects of the management process. Hence, the major emphasis of this chapter concerns the SIS as it relates to the decision-making and planning process.

The value of any information element is dependent upon its usefulness and its collectability. In determining whether to include a particular information element in a SIS, library administrators should consider its usefulness in the decision-making and planning process in relationship to its difficulty or cost of collection. Once the usefulness and the difficulty or cost of obtaining the information have been ascertained, a benefit-cost analysis can be performed to determine the priority of inclusion into the SIS. For example, information of marginal usefulness and requiring a high cost for collection would be assigned a low priority for inclusion into the SIS. On the other hand, information which are exceedingly useful and readily available, would be assigned a high priority for inclusion. Other information elements at neither of these extremes may be evaluated from a benefit-cost point of view to determine their priority for inclusion.

One of the attributes resulting from the development and employment of decision models of the type presented in this dissertation involves the definitiveness with which the data elements essential to the solution of the decision models are specified. Thus, the evaluation of the information elements required for the implementation of these and other decision models represents a convenient and reasonable approach for determining which information elements should be included in a SIS. Philip Morse, commenting on the relationship between analytical decision models and an information system states:

> It is the author's belief, based upon discouraging experience, that neither the computer experts nor the librarian (for different reasons) really know what data would be useful for the librarian to have collected, analyzed and displayed so he can make decisions with some knowledge of what the decision implies. What is needed before the computer designs are frozen is for models, of the sort developed in this book, to be played with, to see which of them would be useful and to see what data are needed and in what form, in order that both models and computers can be used most effectively by the librarian.[2]

The usefulness of a particular data element, which is utilized in a decision model, might be evaluated in terms of:

1. its contribution to the decision which is to be made
2. the contribution of the decision, when carried out, to the performance of the library
3. the sensitivity of the decision as a function of the data
4. the sensitivity of the contribution of the decision, when effected, to the performance of the library
5. multiple uses of the data in other decision areas.

For example, in evaluating the data element, number of circulation uses of documents in a particular subject area, as applied to the decision of determining the optimal number of new titles to purchase, the following statements regarding usefulness seem relevant. This data element plays a major role in determining the optimal number of new titles to purchase. The decision regarding the number of new titles to purchase has a major effect upon the document exposure performance measure. The optimal number of new titles to purchase is sensitive to varying circulation rates. On a short term basis, document exposure is probably not highly sensitive to the number of new titles purchased. Finally, this data element is useful in other decision areas.

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[2] Philip Morse, Library Effectiveness: A Systems Approach, p. 141.

The difficulty or cost of collecting data for a particular data element, used in a decision model, might be evaluated in terms of:

1. its availability in the desired form as a by-product of library operations or from other sources
2. its availability in raw form requiring sampling or other techniques to obtain the data in desired form
3. requirements to collect and synthesize the data, often utilizing sampling techniques.

An example of data which are by-products of normal library operation is circulation use. A count of the number of titles in a particular subject area obtained from the shelf list represents an example of the second type. Finally, an example of a data element requiring both collection and synthesis, is the average length of time a user consults a document within the library. Furthermore, in each of the three cases there is sometimes an additional cost associated with the implementation of the data collection activity. For example, to obtain circulation use data per subject area, initially, certain procedures must be developed if performed manually and certain programs written and debugged if performed by a computer.

In some cases desired data which are difficult to collect can be estimated as a function of a more readily available data element. For example, in-library use of books was shown to be a function of circulation use. Using this relationship, estimates of in-library use can be made as related to circulation use data.

Hence, given an indication of the relative usefulness and an indication of the relative difficulty or cost of collection of information elements, benefit-cost ratios could be computed. Utilization of these ratios provides a basis for determining which elements to include in the SIS.

Finally, we alluded to a persuasive or influencing function of information. Here we refer to the use made of information in the management process for substantiating or justifying a position. An example of this use might occur in the negotiation process for determining the library's budget. In this case the library administrator might use budgetary figures for libraries at universities of equal standing in an attempt to convince university administrative officials of the need for additional library funding. Since we have not explored this aspect of the management process, a consideration of the information elements relative to this is not included in this research. This is not to imply that this facet of the management process is unimportant. Doubtless, this is a very important area and deserving of future research. We conjecture that the concepts of usefulness and cost or difficulty of collection are equally applicable to ascertain the value of information.

## The Statistical Information System

The composition of an individual library's statistical information system is determined as a function of the level of performance of other programs of the library. That is to say, if the benefit-to-cost ratio of including an additional data element in the SIS exceeds the benefit-to-cost ratio resulting from expending an equivalent amount of funds in any other program, then the element should be included. If the benefit-to-cost ratio of including an additional data element in the SIS is less than the benefit-to-cost ratio resulting when an equivalent amount of funds are expended in another program, the data element should not be included. Since the optimal number of data elements included in the SIS for any particular library is a function of the level of performance of that particular library's programs, the optimal composition of the SIS for different libraries will differ.

We do not imply that the determination of the optimal composition of an individual library's SIS is an easy task. In fact, this is an extremely difficult task. Not only is it difficult to estimate a benefit-to-cost ratio for including an individual autonomous data element, but it is even more difficult to estimate this ratio if it is dependent upon the inclusion of other data elements. Furthermore, in some instances, data for a particular data element can be estimated as a function of another data element with a fair degree of accuracy. If it is less costly to pursue this course of action, it is important to ascertain the tradeoff between accuracy of the estimate and the cost savings. Therefore, the process of selecting the data elements to be included in the SIS requires some rather intensive and difficult evaluation. This task is beyond the scope of this research.

However, as a first step in the development of a library SIS, we enumerate the information elements necessary for the implementation of the decision models developed in this dissertation. For each information element we indicate the recommended frequency of collection, method of collection, source and the decision model(s) for which it is relevant. These information elements appear as Appendix H.

The information elements included in this appendix represent the minimum set required to employ and utilize the decision and planning models developed in this dissertation. Data used to verify the assumptions of the models are not included in this listing. The information elements are specified according to the smallest unit for which information is required. For example, a measure of the number of titles held is specified on a per subcollection per subject area basis as required for use in the Title Selection Model. However, a measure of the number of titles held on a per subject area basis is required in the Circulation Demand Model. In this case, data are collected for units defined by subcollections of subject areas. The data relevant to the Circulation Demand Model can then be obtained by aggregating over subcollections.

Cetain data are recommended to be collected on a once every-two-year basis. These data are used to verify estimates of parameters or to provide new estimates of parameters if the new estimates prove to differ significantly from the old ones. The choice of a two year period is somewhat arbitrary. In general, we do not expect the parameter values to differ significantly from one year to the next. On the other hand we might expect them to change significantly over a five-year period. Thus, selection of a two-year period represents a reasonable compromise. Once historical data are available, this choice can be re-evaluated.

Information elements for which data are designated to be collected every two years comprise the majority of the items in the list. Thus a majority of the elements necessary to utilize the models developed in this dissertation need be collected only once every two years. Further data for the information elements designated to be collected on a continuous basis should be readily obtainable as a by-product of the library administrative process. Data for these information elements are expecially easy to obtain if the library's administrative processes are mechanized. For example, circulation data collected by subcollection and subject area should be readily obtainable as a by-product of the circulation control system. A count of the number of new titles added and weeded should be readily obtainable as a by-product of the library's acquisition system.

Finally, certain tasks require a good deal of initial effort but relatively little effort once the system is in operation. For example, the determination of the appropriate D.D. and L.C. Classification Numbers corresponding to the subject areas and the determination of estimates of the number of titles per subject area are tasks requiring a considerable amount of initial effort, but which require little additional effort in subsequent periods.

Based upon our experience in collecting data for a library which does not currently employ mechanized systems, we conclude that the data collection efforts required to utilize the decision models presented in this dissertation are not unreasonable. The efforts required to obtain these data are reduced considerably when these models are applied to a library which utilizes mechanized systems.

## Summary

In this chapter we discussed the value of information as related to the decision-making, planning and persuasive aspects of management. In this regard the facility of models to specify potential information elements for inclusion in a SIS was discussed. The concepts of usefulness and cost or difficulty of collection were presented as a means for determining the relative value of information. Employment of these concepts permits the use of benefit-cost analysis techniques to determine which elements should be included in the SIS. Finally, the data elements relevant to the decision models developed in this research were enumerated along with the recommended frequency of collection, method of collection, source, and use.

# CHAPTER VII

## SUMMARY AND DIRECTIONS FOR FUTURE RESEARCH

### Summary

The research reported on in this dissertation represents an attempt to develop an improved system for the management of libraries. This management system provides a rational framework for library administrators to formulate plans and make decisions which lead to an effective allocation and utilization of scarce resources. The system is designed to be sensitive to the differing structures and compositions peculiar to different universities so as to be applicable to any university library. Innovations in library technology should not necessitate major changes in this system. Finally, modern management techniques, in particular, analytical decision models, are readily integratable into this system.

The development of this management system began with an exploration of what the library is attempting to achieve. After an investigation of the hierarchy of library objectives, we concluded that the one objective which satisfied a set of specified criteria was to maximize the expected future exposures of university community *members to documents given current and anticipated future expenditures.* A measure of performance utilizing the concept of item-use-days was developed to evaluate the degree of attainment of this objective. Finally, the difficulties encountered in constructing an effectiveness measure to make year-to-year or interlibrary comparisons were discussed.

The Planning-Programming-Budgeting System was presented as a rational framework for the management of libraries. Library activities were classified according to four different dimensions. A typical program structure was enumerated utilizing these dimensions. Benefit-cost analysis was discussed as a means for determining an effective allocation for the library's resources. It was shown that resources are allocated and utilized in the most effective manner when the marginal benefit-to-cost ratios for all elements of all programs are equal.

Analytical models were developed to provide guidance to library administrators in the decision-making and planning process for selected subprograms. In the construction of these models relevant variables and factors were identified and pertinent relationships determined germane to each subprogram. The decision models were solved to yield marginal benefit-to-cost ratios which relate to the overall measure of library performance. Decision models were constructed to provide guidance to determine the number of multiple copies of a title to purchase for a subject area, the number of new titles to purchase for a subject area and the number of copies of a title to place on reserve. In addition, a model was constructed to predict the demand rate for circulation use of titles as a function of the circulation use rate and measures of the teaching and research activities of the university for each subject area.

These models were then tested and verified and relevant parameters estimated for the Lippincott Library. For each model an example of its use was demonstrated and a sensitivity analysis performed.

Finally, guidance was offered for the development of a statistical information system to support the decision-making and planning process. In this regard criteria were presented which can be used to evaluate various information elements for inclusion in the SIS.

## Directions For Future Research

A logical extension of this research entails the actual implementation of the management system presented in this dissertation.

There are many problems to be worked out prior to the actual application of this research. In this regard the need is apparent for a systems analysis, management science, operations research or similar named capability to implement and operate a management system of the type developed in this dissertation. This need arises, in part, from the requirement for specialized talents to assimilate and employ sophisticated management techniques. Another factor, contributing to this need, stems from the reality that most library administrators have barely sufficient time to cope with everyday tasks and problems. This leaves insufficient time available to devote to the numerous tasks which must be accomplished and problems which must be solved associated with the implementation of a new management system. This does not imply that library administrators need not devote a great deal of time and effort to the implementation of a new system. Indeed, the success of a new management system depends upon a high degree of involvement on the part of the library administrators. In addition, the employment of a management system of the type devised in this dissertation requires a computer capability. This capability is essential to the solution of the decision models, as well as for data manipulation and analysis.

The amount of effort and resources devoted to the employment of modern management techniques is a decision or controllable variable to the library administrator. Doubtless, up to a point, increased efforts and resources committed to the employment of these techniques manifest themselves in terms of increased library effectiveness. This occurs as the result of the library's ability to make improved plans and decisions. However, this increase in effectiveness is accompanied by an increase in costs. This dilemma presents the library administrator with yet another problem requiring a solution. The question to be answered concerns the amount of effort and resources to commit to the implementation and operation of an improved library management system consistent with the maximization of overall library effectiveness. This problem is deserving of further research.

We conjecture that the basic structure of the management system developed in this research is equally applicable to the management of college libraries. Future research is recommended to determine the feasibility and benefits of such application.

The problems associated with the determining of a measure(s) of effectiveness for making comparisions from year to year for the same library or between libraries were left unresolved in this research. Further research is recommended to develop a measure(s) which can be utilized to make these comparisons.

There is a need for future research to develop and test additional decision models of the type developed in this dissertation. These decision models are central to the management system and to the specification of the data elements which comprise the statistical information system. Examples of areas for further model development include: document identification, periodical selection and personnel utilization. In addition, efforts are needed to test and improve upon current decision models.

Paralleling the need to develop additional and improved decision models is a need to learn more regarding the behavior of library users. For example, little is known of user behavior regarding the use made of borrowed documents, browsing and bibliographic searchs. Further research is needed in this area.

The major thrust of this research centered upon the development of an improved management system to allocate and utilize a given amount of resources in an effective manner. Little was discussed regarding the persuasive aspects of the management process which is designed to increase the amount of resources available to the library. The library's major source of additional resources is the university which yearly allots an amount of funds. The amount of funds allotted by the university each year is determined via negotiations in the budgetary process. This amount of funds is an important determinant of the library's exposure rate. Doubtless, there are many factors involved in the negotiations which determine whether or not the library receives an equitable allotment. An "equitable" allotment represents that amount of university funds allocated to the library such that the marginal benefit-to-cost ratios for all university programs (including the library) are equal. Thus, an analysis of the factors which are significant in determining the amount of funds allotted to the library is of vital importance to the library and constitutes an area of important research.

Finally, it is clear that considerable additional research is required in the design, testing, and implementation of a library statistical information system to support management decision making and planning.

# APPENDIX A

## UNIVERSITY SUBJECT CATEGORIES AND SUBJECT AREAS

| Subject/Department Categories | Subject Areas |
|---|---|
| Accounting | Financial |
| | Managerial |
| Anthropology | Ethnography |
| | Archaeology |
| | Physical Anthropology |
| | Cultural Anthropology-Culture Theory |
| | Linguistic Anthropology |
| Architecture | History-Theory |
| | Design-Construction |
| Astronomy | History |
| | Spherical-Practical |
| | Universe Studies-Cosmology |
| Biological Science | Cellular |
| | Molecular |
| | Organism-Development |
| | Evolution |
| | Ecology |
| | Behavior |
| | Biophysics |
| | Biochemistry |
| Chemical Engineering | Reaction |
| | Design |
| | Transport Processes |
| Chemistry | Physical |
| | Inorganic |
| | Organic |
| | Crystallography |

| | |
|---|---|
| City and Regional Planning | City Design-City Appearance |
| | Regional Planning-Studies and Approaches |
| | Regional Planning-Implementation |
| | Urban Management and Problems |
| | Social Policy Planning |
| Civil Engineering | Structural Mechanics |
| | Transportation Engineering |
| | Water Resources Engineering |
| Classical Archaeology | Classical Archaeology |
| Classical Language and Literature | Greek |
| | Latin |
| Communications | Media |
| | Communications Behavior |
| Economics | Theory |
| | History |
| | Systems |
| | Growth, Development, and Planning |
| | Econometrics Methods |
| | Economic and Social Data |
| | Fiscal Policy and Public Finance |
| | Fiscal and Monetary Theory and Institutions |
| | International Economics |
| | Agricultural Economics |
| | Industrial Organization and Public Policy |
| | Manpower and Labor |
| | Welfare Programs-Urban and Regional Economics |
| Education | Administration |
| | Elementary Instruction |
| | Elementary Curriculum |
| | Secondary Instruction |
| | Secondary Curriculum |

| | |
|---|---|
| Education (continued) | Social Foundations |
| | Psychological Foundations |
| | Psychological Services |
| Electrical Engineering | Power |
| | Circuit and Network Theory |
| | Control Theory |
| | Computer Design |
| | Electromagnetic Fields |
| | Electronics (Classical) |
| | Communications Theory |
| | Solid State Electronics |
| Engineering Mechanics | Dynamic and Vibration Analysis |
| | Stress Analysis |
| English | English Language |
| | Medieval English |
| | Renaissance |
| | Later 17th and 18th centuries |
| | Romantic |
| | Victorian |
| | Recent |
| | American to 1860 |
| | 1860-1915 |
| | Recent |
| Finance | Business Finance |
| | Real Estate |
| | Trusts and Estate Planning |
| Fine Arts | History of Art |
| | Painting-Sculpture-Graphics |
| Folklore | Folklore |
| General Reference Category | General Reference |
| Geography | Geography |
| Geology | Geology |

| | |
|---|---|
| Germanic Language and Literature | Germanic Philology |
| | German-Medieval to 1700 |
| | 18th and 19th centuries |
| | Recent |
| | Scandinavian |
| | Dutch |
| History | Ancient |
| | Medieval European |
| | Modern European |
| | Germany |
| | France |
| | Great Britain |
| | Russia |
| | Near East |
| | Asia |
| | U. S. |
| | Latin America |
| History of Science | History of Science |
| Industrial Management | Business Organization |
| | Operations Management |
| Industrial Relations | Labor Relations |
| | Personnel |
| Insurance | Property and Liability |
| | Life and Health |
| | Pensions and Employee Benefits |
| | Social Insurance |
| International | Marketing |
| | Operations |
| Law | Business Law |
| Library Science | Library Science |
| Linguistics | Linguistics |

| | |
|---|---|
| Marketing | Consumer Behavior |
| | Distribution and Structure |
| | Promotion |
| | Marketing Research |
| | Marketing Management |
| Mathematics | Algebra |
| | Classical Analysis |
| | Functional Analysis |
| | Topology |
| | Geometry |
| Mechanical Engineering | Thermodynamics |
| | Fluid Dynamics |
| | Heat Transfer |
| Metallurgy and Materials Science | Physical |
| | Chemical |
| | Mechanical |
| Music | History-Theory |
| | Composition |
| | Scores |
| Operations Research | Problem Formation |
| | Models |
| Oriental and South Asia Language and Literature | Ancient Near East (pre-Islamic-Egyptian) |
| | Modern Near East (Islamic Culture) |
| | Hebrew |
| | Arabic |
| | Persian |
| | Turkish |
| | South and South East Asia-Classical |
| | South and South East Asia-Modern |
| | East Asia-Chinese |
| | East Asia-Japanese |

| | |
|---|---|
| Philosophy | History and Problems-Ancient |
| | History and Problems-Medieval |
| | History and Problems-Modern |
| | Logic and Philosophy of Science |
| | Metaphysics |
| | Epistemology |
| | Philosophy of Religion and Psychology |
| Physics | Nuclear |
| | High Energy |
| | Solid State |
| | Gravitation and Astrophysics |
| Political Science | Political Theory and Methodology |
| | American Government |
| | Comparative Government |
| | International Relations and Law |
| | Public Administration |
| Psychology | Learning and Motivation |
| | Sensation and Perception |
| | Personality and Psychopathology |
| | Physiological and Comparative |
| | Developmental |
| | Mathematical Psychology-Measurement |
| Public Utilities | Utilities Industry |
| | Public Regulation |
| Religion | Biblical Studies |
| | Christianity |
| | Judaism |
| | Other Religions |
| | Religion in America |
| Romance Language and Literature | Romance Philology |
| | French Language |
| | French Literature-Medieval |
| | 16-17th centuries |

| | |
|---|---|
| Romance Language and Literature (continued) | 18th century |
| | 19th century |
| | Recent |
| | Italian Language |
| | Italian Literature-Medieval |
| | Renaissance |
| | Modern |
| | Spanish Language |
| | Spanish Literature-Medieval |
| | 16th-17th centuries |
| | 18th-20th centuries |
| | Latin American Literature |
| Slavic Language and Literature | Slavic Philology |
| | Russian |
| | Polish |
| | Serbo-Croatian |
| | Baltic |
| Sociology | Sociological Theory |
| | Social Organization |
| | Statistics and Methodology |
| | Demography |
| | Criminology |
| | Comparative Sociology |
| | Sociology of Health and Welfare |
| | Political Sociology |
| | Social Psychology |
| Statistics | Theory |
| | Applied |
| Transportation | Transportation Industry |
| | Traffic Management |
| | Public Regulation |

# APPENDIX B

## TYPICAL UNIVERSITY LIBRARY PROGRAM STRUCTURE

PROGRAM SUBPROGRAM ELEMENT COMPONENT TASK

I. To provide user area and furnishings
  A. To provide user area (library unit)
    1. To provide study area
      a. Physical area
        (1) Private study area
        (2) General study area
      b. Environment
        (1) Heating-air-conditioning
        (2) Lighting
      c. Maintenance and Repair
        (1) Janitorial Service
        (2) Repair
    2. To provide user service area to include rest rooms, snack room, telephones, etc.

PROGRAM SUBPROGRAM ELEMENT COMPONENT TASK

- B. To provide user furnishings (library unit)
    - 1. To provide seating
        - a. Desk-chairs (carrels)
        - b. Table-chair combinations
        - c. Lounge chairs
    - 2. To provide other furnishings

II. To provide access to documents within the library

- A. To manage document collections (subject category)
    - 1. To select documents
        - a. Selection mechanism
        - b. Documents
            - (1) Books (subject area)
            - (2) Periodicals-serials
            - (3) Other
    - 2. To acquire documents
        - a. Ordering
        - b. Receiving
    - 3. To classify documents
    - 4. To process documents

| PROGRAM | SUBPROGRAM | ELEMENT | COMPONENT | TASK |
|---|---|---|---|---|
| | | | a. Binding | |
| | | | b. Numbering and pasting | |
| | | 5. To weed and maintain collection | | |
| | | | a. Rebinding and repairs | |
| | | | b. Relocating | |
| | | | c. Discarding | |
| | | 6. To store documents | | |
| | | | a. Shelving | |
| | | | b. Physical area | |
| | | | c. Environment | |
| | | | d. Maintenance | |
| | B. To control use and location of documents (library unit) | | | |
| | | 1. Circulation | | |
| | | 2. Loans to other libraries | | |
| | | 3. Periodical control | | |
| | | 4. Reserve service | | |
| | | 5. Reshelving | | |
| | | 6. Security | | |
| | C. To facilitate document use (library unit) | | | |
| | | 1. To provide accessory equipment | | |
| | | | a. Microform readers | |
| | | | b. Audio equipment | |
| | | | c. Visual Equipment | |

PROGRAM SUBPROGRAM ELEMENT COMPONENT TASK

- 2. To provide copy service

III. To provide access to documents in other libraries

- A. To provide inter-library document loan (library)
    - 1. To assist users in requesting documents from other libraries
    - 2. To request, receive, control and return documents borrowed from other libraries
- B. To coordinate and administer inter-library access agreements (library)

IV. To provide aids in identifying and locating documents and to provide information

- A. To provide aids to locate documents within the library (library unit)
    - 1. To provide catalogues
        - a. Determining catalogue entries
        - b. Maintaining catalogue
    - 2. To prepare and post directional signs and explanatory placards
- B. To provide aids to identify existing documents and to locate documents in other libraries (library unit)
    - 1. To manage index collection

PROGRAM SUBPROGRAM ELEMENT COMPONENT TASK

- a. Selection
- b. Acquisition
- c. Classification and cataloguing
- d. Processing
- e. Weeding and Maintenance
- f. Storage

2. To control use and location of indexes
3. To facilitate index use

C. To assist in compiling cooperative inter-library catalogues of holdings (library)

D. To provide personal assistance for document identification and location and to provide information (library unit)

    1. To assist users in performing literature research and in identifying or verifying a specific citation
    2. To provide reference information

V. To promote use of the library

A. To provide publications, advertisements and exhibits (library unit)

    1. To provide publications

        a. User guides
        b. Selected bibliographies

    2. To advertise
    3. To construct exhibits

PROGRAM SUBPROGRAM ELEMENT COMPONENT TASK

B. To provide personal communication with members of population being served (library unit)

    1. To perform liaison visits with faculty
    2. To conduct library tours and classes regarding use of the library

VI. To provide general administrative and support services

A. To provide administrative service (library)

    1. To administer personnel activities
    2. To handle payments, receipts, and accounting requirements
    3. To prepare programs, projects, and budgets
    4. To administer reports
    5. To perform other administrative services as required

B. To provide support services (library)

    1. To provide office supplies
    2. To provide copy and duplication service
    3. To provide internal mail service

C. To provide staff area and office equipment

    1. To provide physical area
    2. To provide environment
    3. To provide maintenance
    4. To provide office equipment

# APPENDIX C

## SAMPLING METHODS

An estimate of the number of titles held in each subcollection of each subject area was obtained by taking a random sample of 200 titles from each subject area. If there were fewer than 200 titles in the subject area, a census was obtained for these titles. The sampling of titles for each subject area was accomplished by selecting cards from the library's shelf list. Titles are listed on cards which are arranged by D.D. and L.C. Classification Numbers in order of increasing magnitude in a shelf list. Each title appears on one or more cards. There is never more than one title per card. On the card(s) corresponding to a particular title are listed the call number, the title, the author, the publication date, and the number of copies of the title. Cards were selected at random from the set of cards in the shelf list identified by the span of L.C. or D.D. call numbers which define a particular subject area. For each of the 200 cards selected, the following data were noted:

1. The number of cards on which the same title appears pertinent to each card sampled.
2. The number of copies of the title listed on the cards in 1.

3. The publication date of the title.

Since the probability of inclusion of a particular title in the sample is dependent upon the number of cards in the shelf list relevant to the title, a variable probability of inclusion factor must be utilized in the estimation procedure.

An estimate of the number of titles held in each subject area proceeds as follows. If we let

$K$ = the total number of cards in the shelf list associated with titles in the subject area (as defined by the L.C. and D.D. call numbers appropriate to the subject area)

$n$ = the number of cards sampled

$k_j$ = the number of cards corresponding to the $j^{th}$ title sampled in the subject area where $j = 1, 2, \ldots, n$

$N$ = the number of titles in the subject area

then
$$\hat{N} = \frac{K}{n} \sum_{j=1}^{n} \frac{1}{k_j}$$

and
$$\mathrm{Var}(\hat{N}) = \frac{1}{n} \sum_{j=1}^{n} \frac{k_j}{K} \left(\frac{K}{k_i} - \hat{N}\right)^{2}$$ [*]

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[*] Des Raj, Sampling Theory, McGraw Hill, Inc., N. Y., 1968, p. 48.

An estimate of the number of titles held in subcollection t of a subject area proceeds in the following manner. If we let

$n_t$ = the number of titles in the sample which are from subcollection t

$P_t$ = the proportion of titles in the subject area which are from subcollection t

then

$$\hat{P}_t = \frac{\sum_{j=1}^{n_t} 1/k_j}{\sum_{j=1}^{n} 1/k_j}$$

The variance of $\hat{P}_t$ can be computed utilizing the following approximate expression.[**]

$$\mathrm{Var}(\hat{P}_t) \doteq \frac{\overline{U}^2}{\overline{V}^2}\left(\frac{\mathrm{Var}(U)}{\overline{U}^2} + \frac{\mathrm{Var}(V)}{\overline{V}^2} - \frac{2\,\mathrm{Cov}(UV)}{\overline{UV}}\right)$$

where

$$U = \sum_{j=1}^{n_t} \frac{1}{k_j}$$

$$\overline{U} = E\left(\sum_{j=1}^{n_t} \frac{1}{k_j}\right) = n_t\, E\left(\frac{1}{k_j}\right)$$

---

[**] See M. Hansen, W. Hurwitz and W. Madow, Sample Survey Methods and Theory, John Wiley, N.Y., 1953, p. 513.

$$V = \sum_{j=1}^{n} \frac{1}{k_j}$$

$$\overline{V} = E\left(\sum_{j=1}^{n} \frac{1}{k_j}\right) = nE\left(\frac{1}{k_j}\right)$$

We observe that

$$\overline{U} = \frac{n_t}{n}\,\overline{V}$$

and that

$$\mathrm{Var}(U) = \left(\frac{n_t}{n}\right)^2 \mathrm{Var}(V)$$

$$\therefore\ \mathrm{Var}(\hat{P}_t) \doteq \left(\frac{n_t}{n}\right)^2 \left(\frac{\mathrm{Var}(V)}{\overline{V}^2} + \frac{\mathrm{Var}(V)}{\overline{V}^2} - \frac{2\ \mathrm{Cov}(UV)}{\overline{U}\,\overline{V}}\right)$$

It follows that $\dfrac{2\ \mathrm{Cov}(UV)}{\overline{U}\,\overline{V}} = \dfrac{2\rho\sqrt{\mathrm{Var}(V)}\ \sqrt{\mathrm{Var}(U)}}{\overline{U}\,\overline{V}}$

$$= \frac{2\rho\sqrt{\mathrm{Var}(V)}\ \sqrt{({}^{n_t}/n)^2\ \mathrm{Var}(V)}}{\left({}^{n_t}/_{n}\right)\overline{V}\,\overline{V}}$$

$$\frac{2\ \mathrm{Cov}(UV)}{\overline{U}\,\overline{V}} = \frac{2\rho\mathrm{Var}(V)}{\overline{V}^2}$$

$$\therefore \ \mathrm{Var}(\hat{P}_t) \doteq 2\left(\frac{n_t}{n}\right)^2 \left(\frac{\mathrm{Var}(V)}{\bar{V}^2}\right)(1 - \rho)$$

Since U is a subset of V, we would expect the value of $\rho$ to be positive. Thus the actual variance of $\hat{P}_t$ is somewhat less than

$$2\left(\frac{n_t}{n}\right)^2 \left(\frac{\mathrm{Var}(V)}{\bar{V}^2}\right)$$

dependent upon the value of $\rho$. Since $\mathrm{Var}(V) = n^2 \ \mathrm{Var}(^1/k_j)$, then the value for $\mathrm{Var}(\hat{P}_t)$ is somewhat less than

$$2\left(\frac{n_t}{n}\right)^2 \ \frac{\mathrm{Var}(^1/k_i)}{(E(^1/k_i))^2}$$

Thus, given that

$N_t$ = the number of titles in subcollection t of the subject area

then, $\hat{N}_t = \hat{N} \ \hat{P}_t$

and $\mathrm{Var}(\hat{N}_t) \doteq \hat{N}^2 \ \mathrm{Var}(\hat{P}_t) + \hat{P}_t{}^2 \ \mathrm{Var}(\hat{N})$***

---

*** Idem., where $\mathrm{Cov}(P_t N)$ assumed equal to zero.

An estimate of the proportion of titles in the subject area which have duplicate copies proceeds in a similar fashion. If we let

$n_D$ = the number of titles in the sample for which there are 2 copies

$P_D$ = the proportion of titles in the subject area for which there are duplicate copies

then,

$$\hat{P}_D = \frac{\sum_{j=1}^{n_D} {}^1/_{k_j}}{\sum_{j=1}^{n} {}^1/_{k_j}}$$

and $$\mathrm{Var}(\hat{P}_D) = 2\left(\frac{n_D}{n}\right)^2 \left(\frac{\mathrm{Var}({}^1/k_j)}{(E({}^1/k_j))^2}\right)(1 - \rho)$$

The variance of $P_D$ is somewhat less than

$$2\left(\frac{n_D}{n}\right)^2 \left(\frac{\mathrm{Var}({}^1/k_j)}{(E({}^1/k_j))^2}\right)$$

depending on the value of $\rho$ (which we also expect to be positive).

In a similar manner the proportion of titles for which there are triplicate, etc. copies can be estimated.

In these analyses an unbiased estimate of $E(^1/k_j)$ is

$$Y = \frac{1}{n} \sum_{j=1}^{n} \frac{1}{k_j}$$

and an unbiased estimate of Var $(^1/k_j)$ is

$$\frac{1}{n} \sum_{j=1}^{n} \frac{k_j}{K} \left( \frac{1}{k_j} - Y \right)^2 \text{****}$$

---

****Des Raj, Sampling Theory, p. 48.

# APPENDIX D

COMPARISON OF VALUES OBTAINED BY NUMERICAL INTEGRATION OF EXACT EXPRESSION WITH THE VALUES OBTAINED BY SOLUTION OF APPROXIMATE POLYNOMIAL EXPRESSION FOR THE COMPONENTS OF THE CIRCULATION DEMAND MODEL.

| EXACT EXPRESSION | APPROXIMATE EXPRESSION |
|---|---|
| $I_1 = \frac{1}{\bar{D}} \int_0^\infty \left( \frac{\lambda^2/\mu}{1+\lambda/\mu} \right) e^{-\lambda/\bar{D}} \, d\lambda$ | $I_1' = \frac{2\bar{D}^2}{\mu} - \frac{\bar{D}^3}{3\mu} + \frac{3\bar{D}^4}{5\mu^2} - \frac{3\bar{D}^5}{10\mu^3}$ |
| $I_2 = \frac{1}{\bar{D}} \int_0^\infty \left( \frac{\lambda^3/2\mu^2}{1+\lambda/\mu+\lambda^2/2\mu} \right) e^{-\lambda/\bar{D}} \, d\lambda$ | $I_2' = \frac{3\bar{D}^3}{\mu^2} - \frac{2\bar{D}^4}{3\mu^2} + \frac{11\bar{D}^5}{10\mu^3} - \frac{2\bar{D}^6}{5\mu^4}$ |
| $I_3 = \frac{1}{\bar{D}} \int_0^\infty \left( \frac{\lambda/\mu}{1+\lambda/\mu} \right) e^{-\lambda/\bar{D}} \, d\lambda$ | $I_3' = \frac{\bar{D}}{\mu} - \frac{2\bar{D}^2}{\mu^2} + \frac{3\bar{D}^3}{\mu^3}$ |
| $I_4 = \frac{1}{\bar{D}} \int_0^\infty \left( \frac{\lambda/\mu+\lambda^2/\mu^2}{1+\lambda/\mu+\lambda^2/2\mu^2} \right) e^{-\lambda/\bar{D}} \, d\lambda$ | $I_4' = \frac{\bar{D}}{\mu} + \frac{3\bar{D}^2}{\mu^2}$ |

| $\bar{D}$ | $I_1$ | $I_1'$ | $I_1-I_1'$ | % DIFFERENCE | $I_2$ | $I_2'$ | $I_2-I_2'$ | % DIFFERENCE |
|---|---|---|---|---|---|---|---|---|
| .4 | .01751 | .01760 | -.00009 | .05 | .00060 | .00061 | -.00001 | 1.67 |
| .8 | .05566 | .05608 | -.00042 | .64 | .00441 | .00444 | -.00003 | .68 |
| 1.2 | .13926 | .13968 | -.00042 | .30 | .01353 | .01371 | -.00018 | 1.33 |
| 1.6 | .23505 | .23383 | +.00122 | .52 | .02955 | .02967 | -.00012 | .41 |
| 2.0 | .35038 | .34499 | +.00539 | 1.54 | .05337 | .05300 | +.00037 | .69 |
| 3.0 | .70910 | .68274 | +.02636 | 3.72 | .14943 | .14434 | +.00509 | 3.40 |
| 4.0 | 1.14827 | 1.09641 | +.05186 | 4.51 | .29964 | .28347 | +.01617 | 5.40 |
| 5.0 | 1.66376 | 1.59695 | +.06691 | 4.02 | .51119 | .48066 | +.03053 | 5.97 |
| 6.0 | 2.21971 | 2.21584 | +.00387 | .17 | .76655 | .77016 | -.00361 | .47 |
| 7.0 | 2.81758 | 2.99771 | -.18013 | 6.39 | 1.06798 | 1.22148 | -.15350 | 14.37 |

| $\bar{D}$ | $I_3$ | $I_3'$ | $I_3-I_3'$ | % DIFFERENCE | $I_4$ | $I_4'$ | $I_4-I_4'$ | % DIFFERENCE |
|---|---|---|---|---|---|---|---|---|
| .4 | .00235 | .00246 | -.00011 | .49 | .02442 | .02436 | +.00006 | .24 |
| .8 | .04298 | .04294 | +.00004 | .09 | .05055 | .05038 | +.00017 | .34 |
| 1.2 | .06186 | .06168 | +.00018 | .29 | .07814 | .07806 | +.00008 | .10 |
| 1.6 | .07958 | .07890 | +.00068 | .86 | .10686 | .10740 | -.00054 | .51 |
| 2.0 | .09618 | .09485 | +.00133 | 1.38 | .13621 | .13841 | +.00220 | 1.62 |
| 3.0 | .13313 | .13068 | +.00245 | 1.84 | .23880 | .22318 | +.01562 | 6.54 |

NOTE: $I_1$ and $I_2$ are components of the Circulation Demand Model in which the number of circulations is employed. $I_3$ and $I_4$ are components of the Circulation Demand Model in which the number of titles out, L, is employed.

# APPENDIX E

## LIBRARY OF CONGRESS AND MODIFIED DEWEY DECIMAL CLASSIFICATION NUMBERS CORRESPONDING TO ACADEMIC SUBJECT AREAS AS USED BY LIPPINCOTT LIBRARY

| SUBJECT CATEGORY | SUBJECT AREA | L.C. CALL # | MODIFIED D.D. CALL # |
|---|---|---|---|
| Accounting | Financial | HF 5601-5686.C7 | 339.73 |
| | | HF 5686.7-5689 | 339.74-339.749 |
| | Managerial | HF 5686.C8 | 339.7-339.7255 |
| | | | 339.7301-339.73512 |
| Computer Science | | QA 74-76.8 | 510.78 |
| | | QA 264-268 | |
| | | TA 345 | |
| | | Z 699-699.5 | |
| Economics | Agric. Econ. | HD 1401-1493 | 330.4 |
| | | HD 1751-2206 | 332.4-332.47 |
| | Econometrics | HB 74 M3 | 330.02 |
| | Econ. & Social Data | HB 849-3700 | 310.015-319 |
| | Fisc. & Monetry Theo | HJ 131-135 | 330.206-330.25 |
| | | HG 220-230 | |
| | Fiscal Pol. & Public Fin. | HJ 9-130 | 336-336.9 |
| | | HJ 136-1999 | |
| | | HJ 2005-5797 | |
| | | HJ 5828-5957 | |
| | | HJ 7451-9995 | |
| | Growth, Dev. & Planning | HB 3711-3840 | 330.3-330.313 |
| | | HC 59.7-710 | 330.33-330.38 |
| | | HD 82-85 | 330.5-330.68 |
| | | HT 101-395 | 339.6902-330.695 |
| | | Z 7164.B91 | 335.1-335.19 |
| | History | HB 75-86 | 330.1-330.17 |
| | | HB 88-97.4 | |
| | | HB 97.6-100 | |
| | | HC 1-59 | |
| | Industries | HD 2951-3570 | 355 |
| | | HD 9000-9999 | 337's |
| | International Economics | HF 1007-1025 | 332.16-332.184 |
| | | HF 1401-4040 | 332.7 |
| | | HG 182-200 | 332.812 |
| | | HG 3881-3883 | 382.6-382.7 |
| | | HJ 6041-6465 | |
| | | HJ 6603-7384 | |
| | Theory | HB 1-73 | 330-330.1 |
| | | HB 151-845 | 330.03-330.09 |
| | | | 330.2-330.205 |

| SUBJECT CATEGORY | SUBJECT AREA | L.C. CALL # | MODIFIED D.D. CALL # |
|---|---|---|---|
| Economics (continued) | Systems | HB 87<br>HB 97.5<br>HB 101-129<br>HM 35<br>HM 211<br>HX 44<br>HX 71-517 | 330.32<br>330.9-330.95 |
| | Welfare Progr. | HV 51-516 | 301.5<br>330.8-330.85 |
| Finance | Banking | HG 1501-3540 | 332.2-332.39<br>332.813-332.92 |
| | Business Finance | HF 5550-5561<br>HG 3701-3880<br>HG 4001-4280<br>HG 4501-6051 | 332.5-332.64<br>332.8-332.81<br>333-333.23<br>333.6<br>334-334.08<br>334.1-334.156<br>382.214 |
| | Money | HG 173-181<br>HG 201-219<br>HG 231-1486 | 332-332.156 |
| | Real Estate | HD 251-1130 | 335-335.08<br>335.3-335.32<br>335.4-335.47 |
| | Trust & Estate Planning | HG 4301-4495<br>HJ 5801-5819<br>KF 731-750 | 347.6-347.8<br>333.5-333.52<br>334.09 |
| Industrial Management | Business Organization | HD 1375-1395<br>HF 5500-5548<br>HF 5565-5599 | 339-339.014<br>339.03-339.3553<br>330.69-330.6901 |
| | Operations Management | HD 1-20.2<br>HD 20.6-69<br>HD 70.76<br>HD 2741-2747<br>HF 5691-5780<br>T 50-60.5<br>T 174-178<br>TK 1811<br>TK 1841<br>TS 155-156 | 339.4-339.67<br>339.75-339.972 |
| Industrial Relations | Labor Relations | HD 5701-7080<br>HD 8045-8942<br>HD 4813-4909 | 331-331.29<br>331.88-331.899<br>331.93-331.95 |

| SUBJECT CATEGORY | SUBJECT AREA | L.C. CALL # | MODIFIED D. D. CALL # |
|---|---|---|---|
| Industrial Relations (continued) | Personnel Relations | HD 7255-7813<br>HF 5548.8-5549.5 | 331.3-331.798 |
| Insurance | Business | HG 8011-8750 | 368.01-368.5 |
|  | Life & Health | HG 8751-8924<br>HG 8930-9245<br>HG 9383-9399 | 368.53-368.537 |
|  | Property & Liability | HG 9301-9343<br>HG 9651-9899<br>HG 9956-9970 | 335.33-335.35<br>368.51-368.52 |
|  | Soc. Insurance | HD 7090-7250<br>HD 7814-7817<br>HG 8925-8929 | 368.55-368.559 |
| International Business |  | HD 69.17<br>HG 136 | 333.4<br>382-382.212<br>382.22-382.26 |
| Business Law |  | HD 7819-7960<br>HF 1201-1400<br>KF 841-2057<br>KF 3176 - KFY | 331.90331.91<br>347.2-347.5<br>382.5 |
| Marketing | Consumer Behavior | HF 5415.3 | 385.1-385.15 |
|  | Management | HF 5415.1 | 386-388 |
|  | Promotion & Dist. | HF 5415.4-5415.7<br>HF 5801-5866 | 385.2-385.7 |
|  | Research | HF 5415.2 | 385.015 |
|  | Theory | HF 5415 | 385-385.01<br>385.02-385.09 |
| Statistics & Operations Research | Operations Research | HD 20.3-20.5<br>Q 500-510<br>QA 269-270<br>QA 402<br>TA 168 | 339.015<br>519.9 |
|  | Statistics | HA 1-35<br>QA 273-277 | 310.01<br>519-519.7 |
| Transportation & Public Utilities | Traffic Mgmt. | HE 331-373<br>HE 593-597<br>HE 1821-2591<br>HE 2701-4211 | 380.1<br>380.81<br>380.83-380.838 |

| SUBJECT CATEGORY | SUBJECT AREA | L. C. CALL # | MODIFIED D. D. CALL # |
|---|---|---|---|
| | Transportation Ind. | HE 1-192 | 380-380.09 |
| | | HE 201-300 | 380.6-380.724 |
| | | HE 381-581 | 380.729-380.808 |
| | | HE 599-945 | 380.8384-380.8751 |
| | | HE 1001-1050 | 380.9-380.973 |
| | | HE 1601-1813 | |
| | | HE 4291-5881 | |
| | | HE 5889-5999 | |
| | Transportation Public Regulation | HE 193-199.9 | |
| | | HE 585-587 | 380.2 |
| | | HE 951-953 | 380.725-380.728 |
| | | HE 1051-1081 | 380.82 |
| | | HE 4221-4281 | 380.8752 |
| | | HE 5886 | 380.99 |
| | | KF 2161-2849 | |
| | Utilities Ind. | HD 1691-1701 | 334.2-334.508 |
| | | HD 2763-2769 | 334.51-334.806 |
| | | HD 3840-4730 | 380.89-380.892 |
| | | TC 5 | |
| | | TC 415-524 | |
| | | TC 801-924 | |
| | | TD 201-500 | |
| | | TK 1191-1541 | |
| | | TK 9001-9153 | |
| | | TP 700-739 | |
| | Utilit. Reg. | HE 7645-9680 | 334.509 |
| | | KF 2076-2140 | 334.806 L |
| | | | 334.1-347.15 |

# APPENDIX F

## QUESTIONNAIRES

### QUESTIONNAIRE #1

Example of the type of questionnaire administered to the faculty members of the Department of Economics to assertain their primary and secondary areas of research interest. Similar questionnaires were distributed to the members of other departments with the appropriate listing of subject areas.

### QUESTIONNAIRE #2

Sample of the questionnaire administered to the professor to determine the assigned reading level of a reserve title used for a particular course.

QUESTIONNAIRE #1

*UNIVERSITY of PENNSYLVANIA*
PHILADELPHIA 19104

*Wharton School of*
*Finance and Commerce*

DEPARTMENT OF STATISTICS
AND OPERATIONS RESEARCH

We are conducting a research investigation whose goal is the design and establishment of a national model for a system of library statistical data to provide a framework for rational planning and decision making concerning library operations and resources. To complete this study, we need some information from you as specified on the enclosed questionnaire. All questions refer to the Spring semester of 1970. If you were not at the University of Pennsylvania at that time, please ignore this questionnaire.

Thank you for your cooperation.

Sincerely yours,

Michael Bommer

Michael Bommer
Research Associate

MB;et
encl.

QUESTIONNAIRE #1

Please indicate your primary area of research interest (in Spring 70) by the number (1). Also indicate your secondary area of interest, if significant, by the number (2); etc.

Economic Theory ____
Economic History ____
Economic Systems ____
Economic Growth, Dev. and Planning ____
Econometrics ____
Economic and Social Data ____
Fiscal Policy and Public Finance ____
Fiscal and Monetary Theory ____
International Economics ____
Agricultural Economics ____
Industries ____
Welfare Programs ____
(other) ________________________ (area) ____

PLEASE RETURN TO: Library Project/Dept. of Stat. & O.R.
Wharton School
E-220 Dietrich Hall
University of Pennsylvania
Phila., Pa. 19104

10/5/70
WD:et

QUESTIONNAIRE #2

*UNIVERSITY of PENNSYLVANIA*
PHILADELPHIA 19104

*Wharton School of*
*Finance and Commerce*

DEPARTMENT OF STATISTICS
AND OPERATIONS RESEARCH

May 12, 1971

Dear Professor

We are developing a model to assist in the decision making process for providing effective library reserve service. Your completion of this form would be most helpful. For each of the books listed below which were on reserve during the Spring Semester, 1970, please indicate how the books were used in your course by entering a (1), (2), (3), (4), (5), or (6) in the column labeled "Assigned Reading Level." If there was more than one section of your course, and if the assigned reading level for a book varied from section to section, please indicate to the best of your knowledge how the book was utilized in the majority of the sections.

(1) All students were required to read more than one article or chapter in the book

(2) All students were required to read one article or chapter in the book

(3) Some students were required to read more than one article or chapter in preparing specific assignments

(4) Some students were required to read one article or chapter in preparing specific assignments

(5) Not required reading for this course

(6) Other (please specify)

| BOOK | COURSE | ASSIGNED READING LEVEL |
|---|---|---|

Please return this questionnaire in the envelope provided. Thank you for your cooperation.

Cordially,

Michael Bommer
Michael Bommer
Research Associate

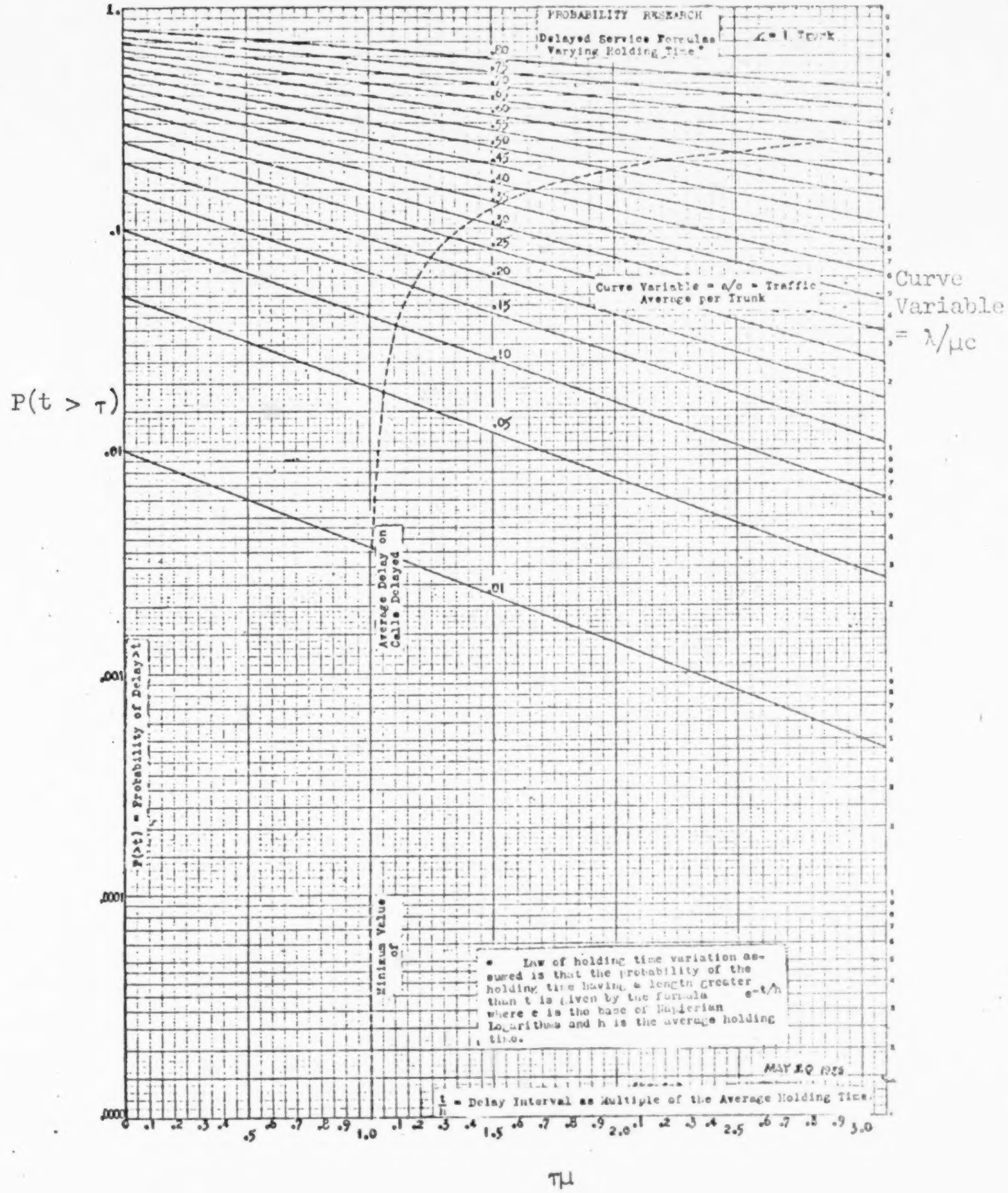
MB:et

# APPENDIX G

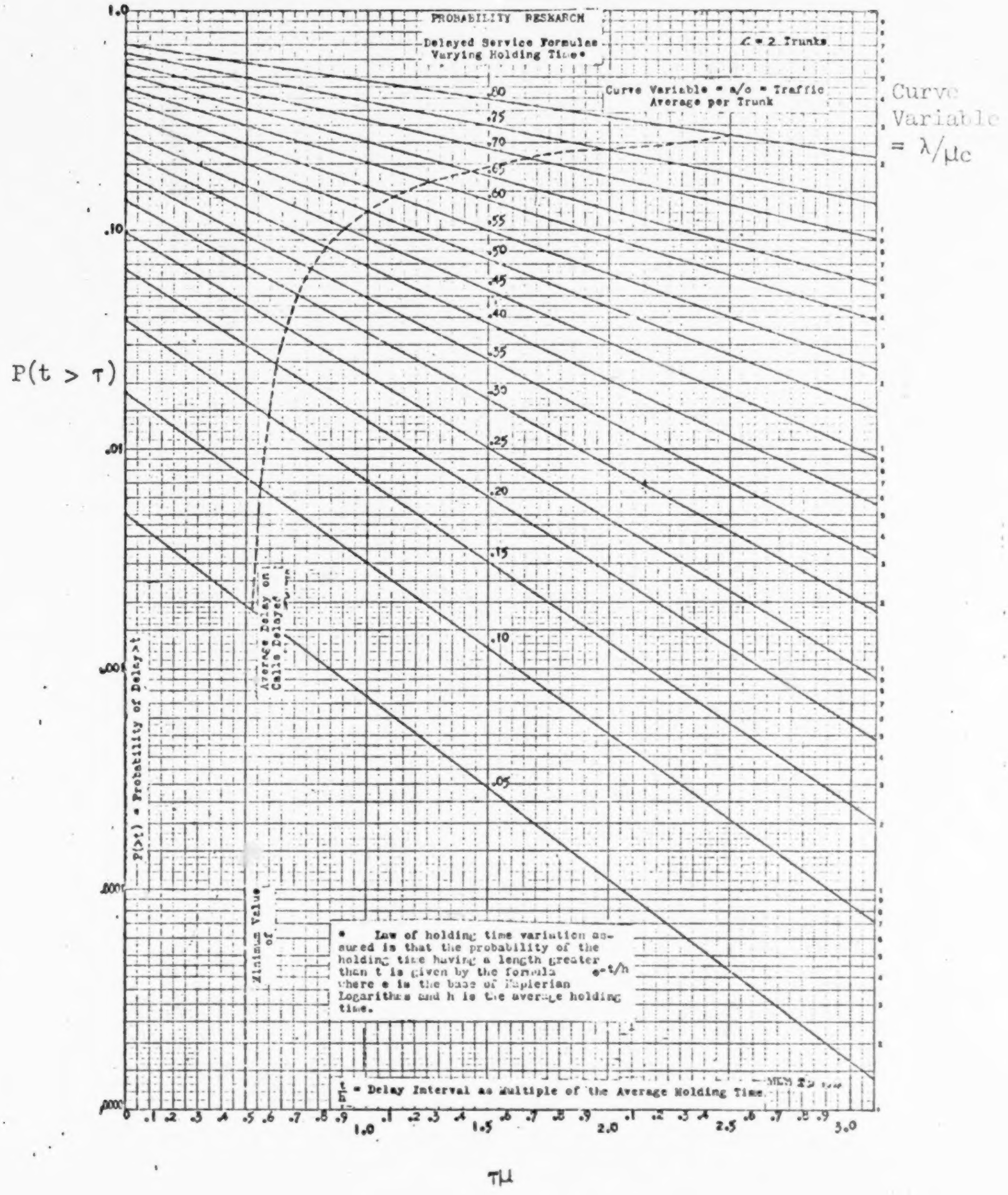
## GRAPHICAL SOLUTION OF EQN (4.65) FOR DETERMINING THE PROBABILITY A USER WILL HAVE TO WAIT LONGER THAN $\tau$ WEEKS, $P(t > \tau)$, TO OBTAIN A COPY OF A TITLE ON RESERVE

NOTE: The following 7 graphs are reproduced from Edward Molina, "Application of the Theory of Probability to Trunking Problems," The Bell System Technical Journal, 6:475-481 (1927).

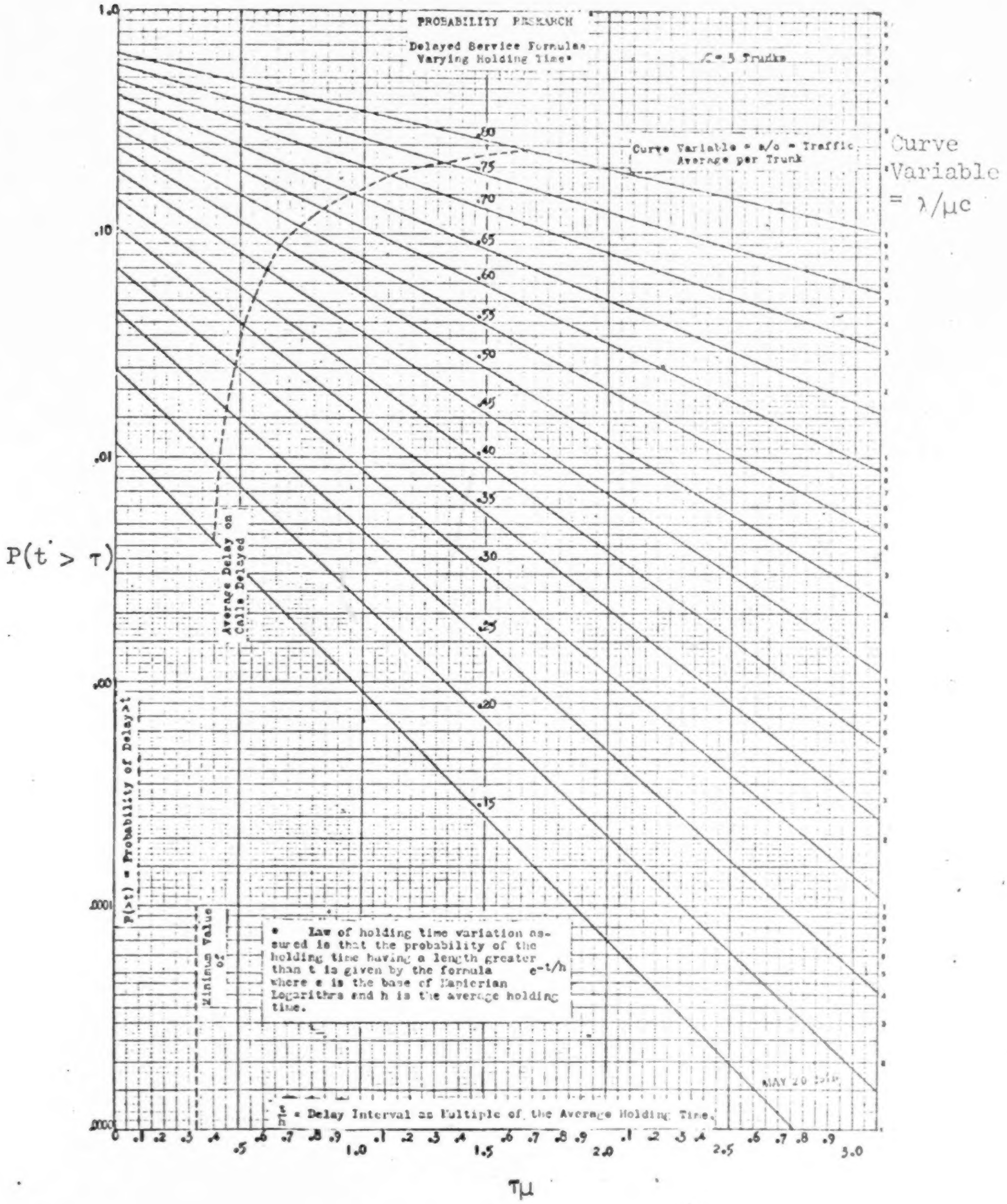
| Symbolism Used in This Dissertation | Corresponding Symbolism Used in Molina's Graphs |
|---|---|
| $P(t > \tau)$ | $P(> t)$ |
| $\tau\mu$ | $t/h$ |
| $\lambda/\mu c$ | $a/c$ |



* Law of holding time variation assumed is that the probability of the holding time having a length greater than t is given by the formula $e^{-t/h}$ where e is the base of Napierian Logarithms and h is the average holding time.



* Law of holding time variation assured is that the probability of the holding time having a length greater than t is given by the formula $e^{-t/h}$ where e is the base of Napierian Logarithms and h is the average holding time.



PROBABILITY RESEARCH

Delayed Service Formulas Varying Holding Time*

c = 3 Trunks

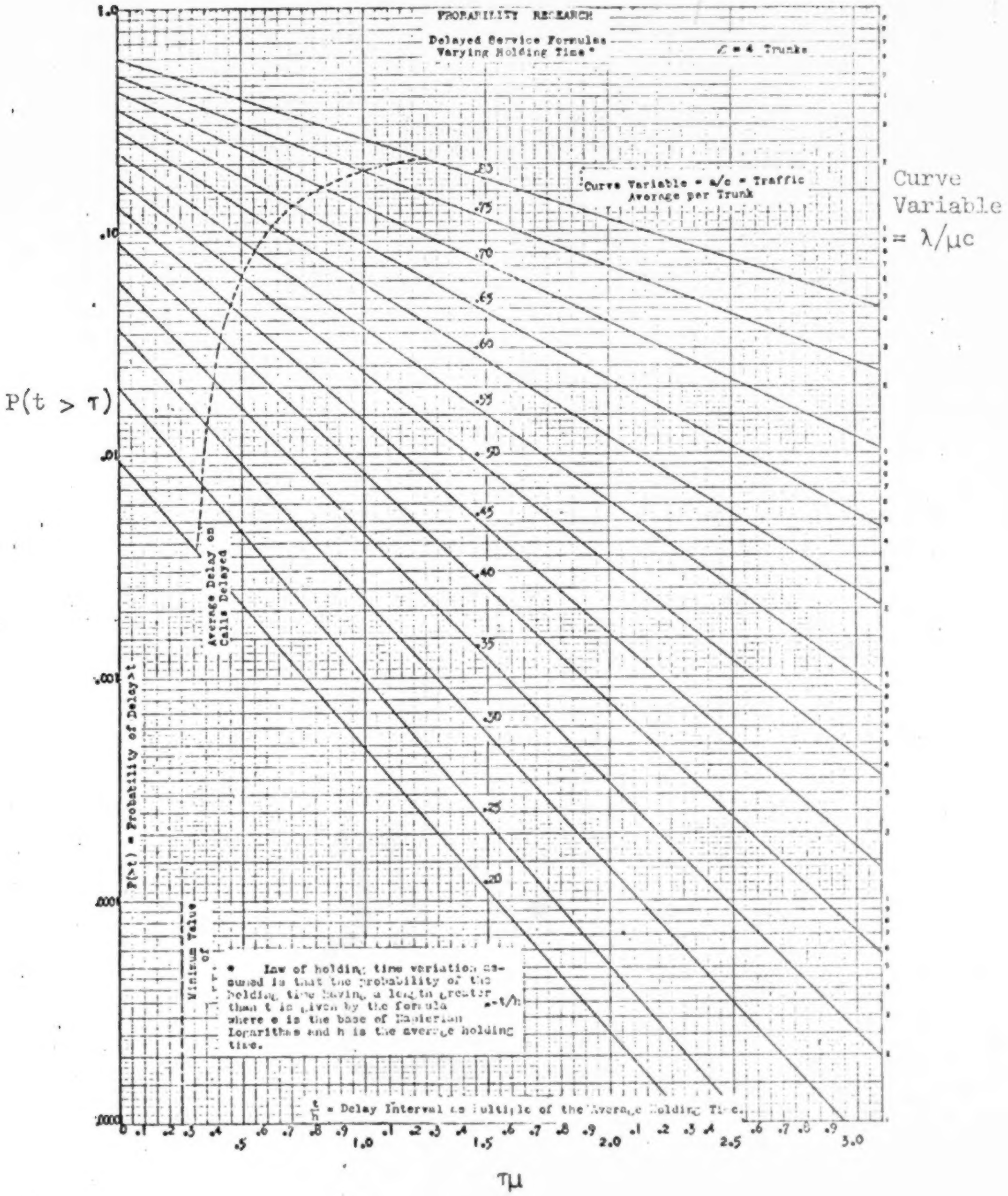
Curve Variable = a/c = Traffic Average per Trunk

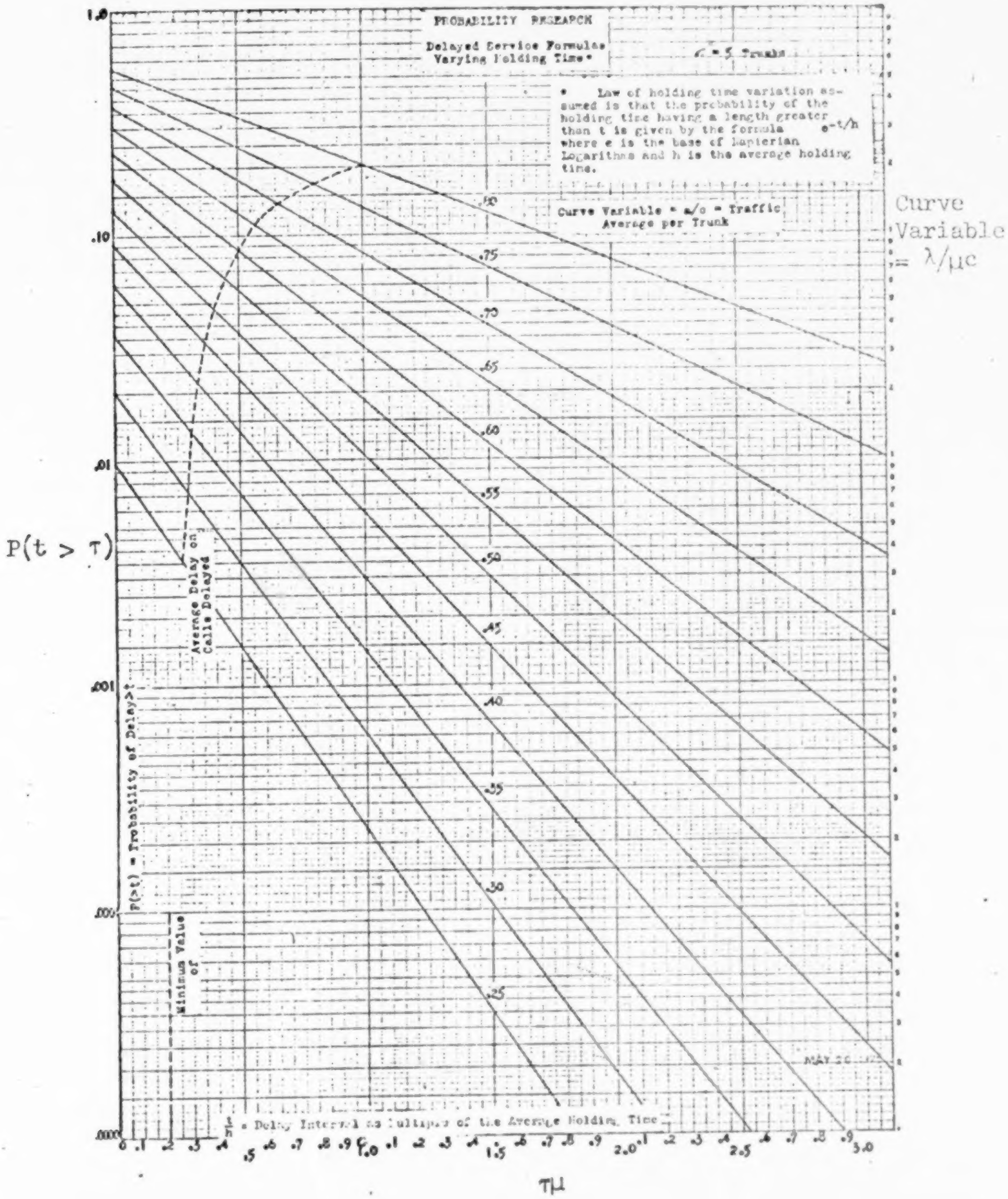
* Law of holding time variation assured is that the probability of the holding time having a length greater than t is given by the formula $e^{-t/h}$ where e is the base of Napierian Logarithms and h is the average holding time.

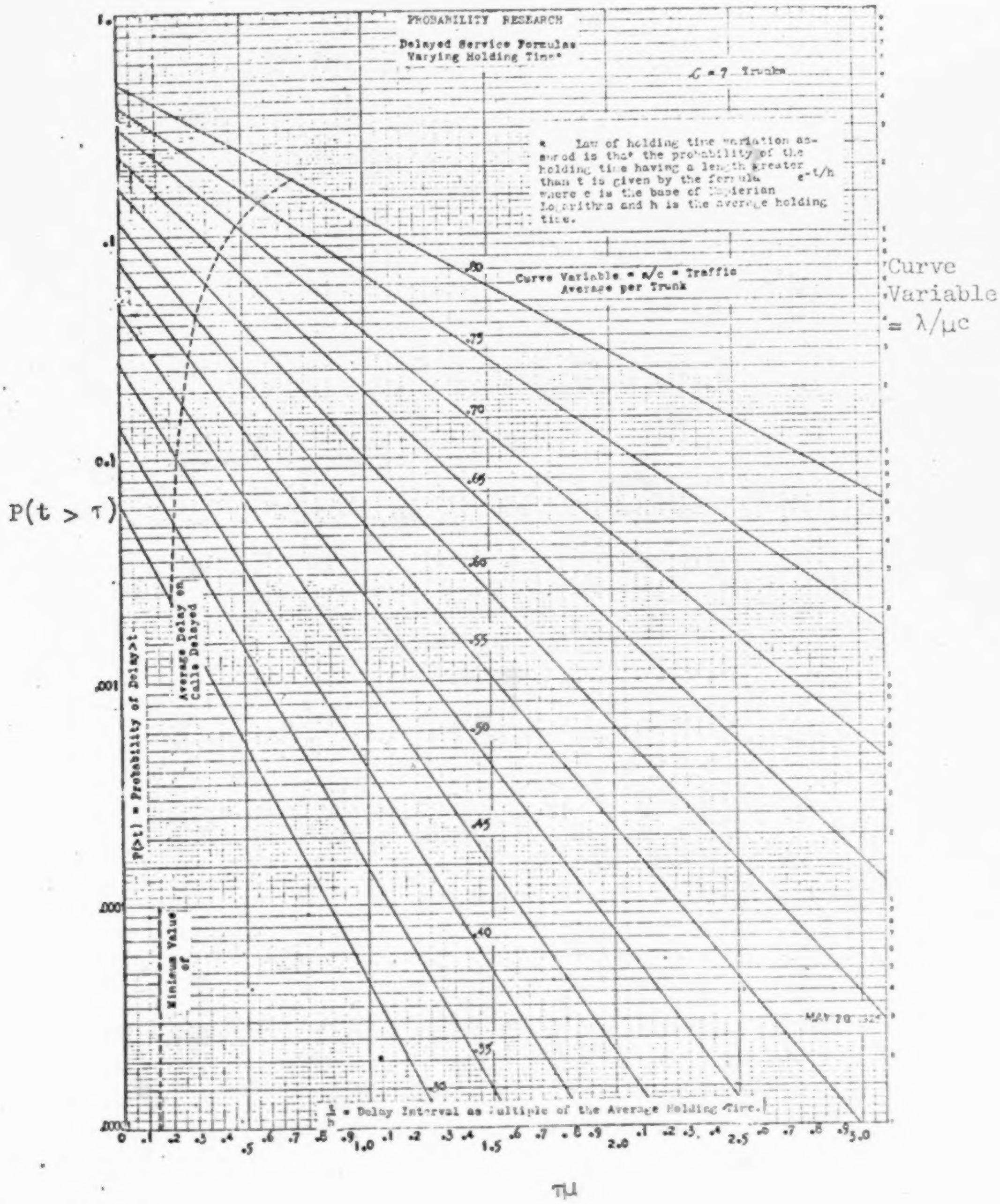
$\frac{t}{h}$ = Delay Interval as Multiple of the Average Holding Time.

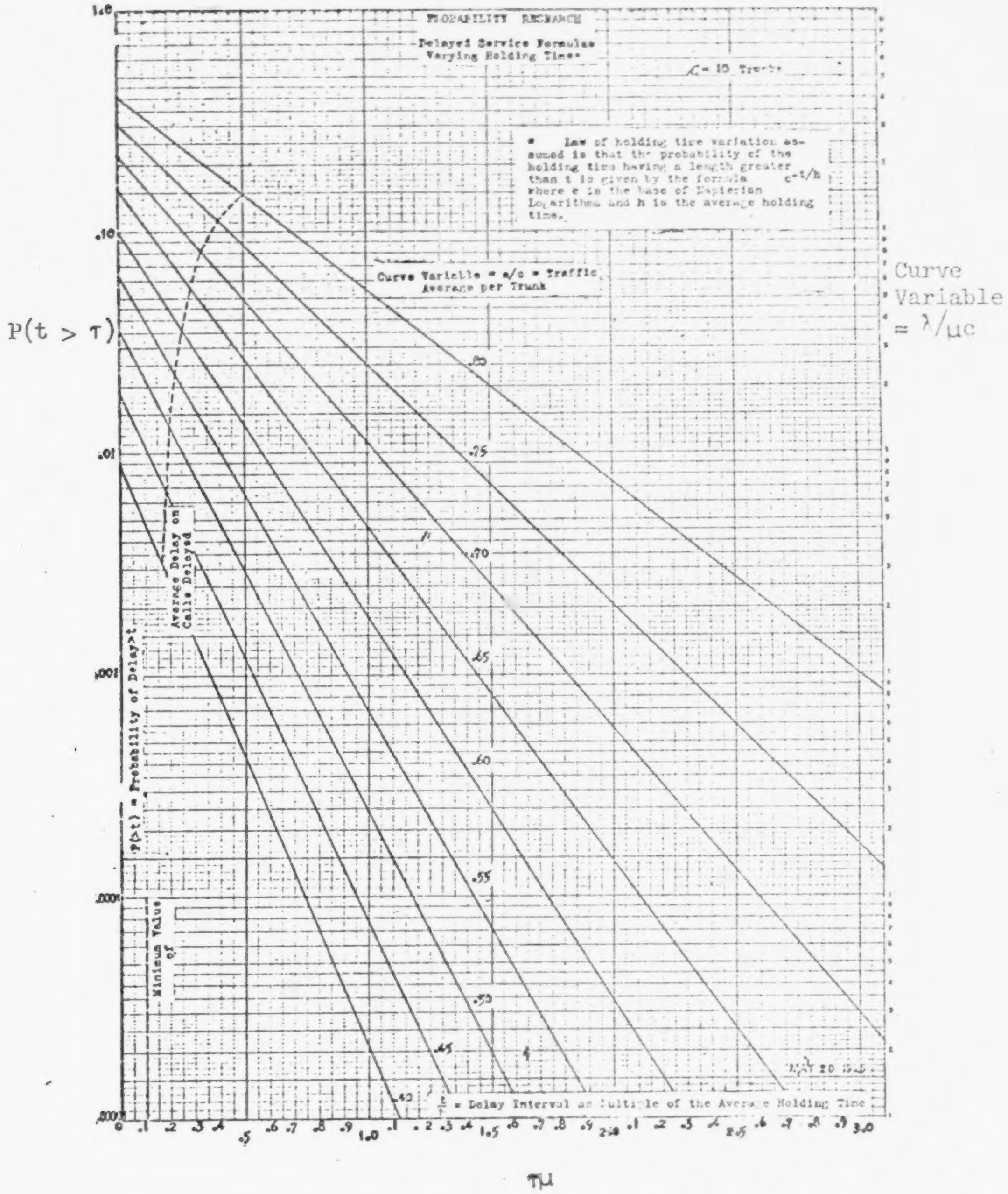
MAY 20 1918

1.0
.10
.01
.001
.0001
.00001
.80
.75
.70
.65
.60
.55
.50
.45
.40
.35
.30
.25
.20
.15
Average Delay on Calls Delayed
Minimum Value of
P(>t) = Probability of Delay>t
P(t' > τ)
Curve Variable = λ/μc
0 .1 .2 .3 .4 .5 .6 .7 .8 .9 1.0 .1 .2 .3 .4 1.5 .6 .7 .8 .9 2.0 .1 .2 .3 .4 2.5 .6 .7 .8 .9 3.0
τμ


PROBABILITY RESEARCH
Delayed Service Formulas
Varying Holding Time*
c = 4 Trunks
Curve Variable = a/c = Traffic Average per Trunk
Curve Variable = λ/μc
.80
.75
.70
.65
.60
.55
.50
.45
.40
.35
.30
.25
.20
Average Delay on Calls Delayed
Minimum Value of
1.0
.10
.01
.001
.0001
.00001
P(>t) = Probability of Delay>t
P(t > τ)
* Law of holding time variation assumed is that the probability of the holding time having a length greater than t is given by the formula e^{-t/h} where e is the base of Naperian Logarithms and h is the average holding time.
t/h = Delay Interval as Multiple of the Average Holding Time.
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
.1
.2
.3
.4
1.5
.6
.7
.8
.9
2.0
.1
.2
.3
.4
2.5
.6
.7
.8
.9
3.0
τμ


PROBABILITY RESEARCH
Delayed Service Formulas
Varying Holding Time*
c = 5 Trunks
* Law of holding time variation assumed is that the probability of the holding time having a length greater than t is given by the formula $e^{-t/h}$ where e is the base of Napierian Logarithms and h is the average holding time.
Curve Variable = a/c = Traffic Average per Trunk
Curve Variable = $\lambda/\mu c$
1.0
.10
.01
.001
.0001
.00001
P(>t) = Probability of Delay>t
$P(t > \tau)$
Average Delay on Calls Delayed
Minimum Value of
.80
.75
.70
.65
.60
.55
.50
.45
.40
.35
.30
.25
$\frac{t}{h}$ = Delay Interval as Multiples of the Average Holding Time
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
.1
.2
.3
.4
1.5
.6
.7
.8
.9
2.0
.1
.2
.3
.4
2.5
.6
.7
.8
.9
3.0
$\tau\mu$
MAY 26


PROBABILITY RESEARCH
Delayed Service Formulas
Varying Holding Time*
c = 7 Trunks
* Law of holding time variation assumed is that the probability of the holding time having a length greater than t is given by the formula $e^{-t/h}$ where e is the base of Napierian Logarithms and h is the average holding time.
Curve Variable = a/c = Traffic Average per Trunk
Curve Variable = $\lambda/\mu c$
.80
.75
.70
.65
.60
.55
.50
.45
.40
.35
.30
Average Delay on Calls Delayed
Minimum Value of
P(>t) = Probability of Delay>t
P(t > τ)
1.
.1
0.1
.001
.0001
.00001
t/h = Delay Interval as Multiple of the Average Holding Time.
0
.5
1.0
1.5
2.0
2.5
3.0
τμ
MAY 20 1925


PROBABILITY RESEARCH
Delayed Service Formulae Varying Holding Time*
c = 10 Trunks
* Law of holding time variation assumed is that the probability of the holding time having a length greater than t is given by the formula $e^{-t/h}$ where e is the base of Napierian Logarithms and h is the average holding time.
Curve Variable = a/c = Traffic Average per Trunk
Curve Variable = $\lambda/\mu c$
P(t > τ)
P(>t) = Probability of Delay>t
Average Delay on Calls Delayed
Minimum Value of
t/h = Delay Interval as Multiple of the Average Holding Time
1.0
.10
.01
.001
.0001
.00001
.80
.75
.70
.65
.60
.55
.50
.45
.40
0
.1
.2
.3
.4
.5
.6
.7
.8
.9
1.0
1.5
2.0
2.5
3.0
τμ

## APPENDIX H

## INFORMATION ELEMENTS

| | Information Element | Source | Recommended Method and Frequency of Collection | Use |
|---|---|---|---|---|
| 1. | Number of titles held per subcollection per subject area ($N_{it}$) | Shelf list | Initially, a one-time sampling of shelf list cards as described in Appendix C; thereafter, a count of the additions and deletions to the shelf list on a continuous basis. | Used in the CD and TS Models |
| 2. | Proportion of titles for which there are duplicate, triplicate, etc., copies | Shelf list | See 1. | Used in CD Model |
| 3. | Number of circulations per subcollection per subject area ($C_{it}$) | Circulation Control System | Count of the number of circulations within each subject area by publication date on a continuous basis. | Used in CD and TS Models and for estimating mean loan period, $1/\mu$ |
| 4. | Number of documents out on loan at a particular point in time ($L$) | Circulation Control System | Count of the number of documents out on loan at a particular time performed once every two years. | Used for estimating $1/\mu$ |
| 5. | Number of documents used in the library per subject area ($H_i$) | Special Collection effort | Count of the number of documents left by users on study tables and desks by subject area; performed for approximately a two-week period once every two years. | Used to estimate factor ($\beta_i$) which relates $H_i$ with $C_i$ |
| 6. | Proportion of titles that have not circulated per subject area ($p_o$) | Circulation record of individual title | Sampling of titles in stacks to determine whether or not document has circulated; performed once every two years. | Used in the CD Model |
| 7. | Actual numbers of students enrolled in courses related to subject areas | Registrar | Courses are classified according to subject areas. The numbers of students enrolled in these courses are ascertained. This is performed for one semester every two years. | Used to estimate parameters in the CD Model |

NOTE: CD - Circulation Demand, TS - Title Selection, MC - Multiple Copy, RS - Reserve Service

| | Information Element | Source | Recommended Method and Frequency of Collection | Use |
|---|---|---|---|---|
| 8. | Projected numbers of students enrolled in courses related to subject areas | University Planning Office or Departmental Offices | See 7. | Used to forecast demand with the CD Model and to determine the optimal number of titles to purchase with the TS Model |
| 9. | Actual numbers of faculty with research interests related to subject areas | Faculty members | Questionnaire administered to faculty once every two years. | Used to estimate parameters in the CD and TS Models |
| 10. | Projected numbers of faculty with research interests related to subject areas | University Planning Office or Departmental Offices | Estimates obtained from appropriate source. | Used to forecast demand with the CD Model and to determine optimal number of titles to purchase with the TS Model |
| 11. | Average cost to purchase an additional copy of a title ($F_1$ and $F_3$) | Financial and accounting records | Estimates of the purchase and processing costs associated with the addition of an extra copy of a title every two years. | Used in the MC and RS Models |
| 12. | Average cost to purchase an additional title ($F_2$) | Financial and accounting records | Estimates of the purchase, classification and processing costs associated with the addition of a new title every two years. | Used in the TS Model |
| 13. | Demand rate data for individual titles on reserve | Reserve Control System | Determine number of demands for each of a sampling of titles on reserve during a semester once every two years. | Used to estimate parameters in RS Model |
| 14. | Number of students enrolled in courses for which reserve titles are assigned as required reading | Pre-registration data and form submitted by professors requesting that a title be placed on reserve | For each title to be placed on reserve, the relevant courses and numbers of enrolled students are determined each semester. | Used to estimate parameters in RS Model and for determining optimal number of copies of a title to place on reserve |
| 15. | Mean time a reserve title is out on loan | Special sample | Determination of the mean use time for titles which are consulted on reserve every two years. | Used in the RS Model |
| 16. | Value of the Library's marginal exposure to cost ratio (U) | Solution of decision Models | A schedule of marginal exposure to cost ratios is generated, associated with various alternative levels of library funding each year. | Used in the MC, TS and RS Models |